## North Carolina Department of Public Instruction

INSTRUCTIONAL SUPPORT TOOLS
FOR ACHIEVING NEW STANDARDS

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4th}\mathrm{ Grade Mathematics • Unpacked Contents
For the new Standard Course of Study that will be effective in all North Carolina schools in the 2017-18 School Year.
This document is designed to help North Carolina educators teach the \(4^{\text {th }}\) Grade Mathematics Standard Course of Study. NCDPI staff are continually updating and improving these tools to better serve teachers and districts.
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## What is the purpose of this document?

The purpose of this document is to increase student achievement by ensuring educators understand the expectations of the new standards. This document may also be used to facilitate discussion among teachers and curriculum staff and to encourage coherence in the sequence, pacing, and units of study for grade-level curricula. This document, along with on-going professional development, is one of many resources used to understand and teach the NC SCOS.

## What is in the document?

This document includes a detailed clarification of each standard in the grade level along with a sample of questions or directions that may be used during the instructional sequence to determine whether students are meeting the learning objective outlined by the standard. These items are included to support classroom instruction and are not intended to reflect summative assessment items. The examples included may not fully address the scope of the standard. The document also includes a table of contents of the standards organized by domain with hyperlinks to assist in navigating the electronic version of this instructional support tool.

How do I send Feedback?
Link for: Feedback for NC's Math Unpacking Documents We will use your input to refine our unpacking of the standards. Thank You!

## Just want the standards alone?

Link for: NC Mathematics Standards

## North Carolina Course of Study - $4^{\text {th }}$ Grade Standards

| Standards for Mathematical Practice |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Operations \& Algebraic Thinking | Number \& Operations in Base Ten | Number \& OperationsFraction | Measurement \& Data | Geometry |
| Represent and solve problems involving multiplication and division. NC.4.0A. 1 <br> Use the four operations with whole numbers to solve problems. <br> NC.4.0A. 3 <br> Gain familiarity with factors and multiples. <br> NC.4.0A. 4 <br> Generate and analyze patterns. <br> NC.4.0A. 5 | Generalize place value understanding for multi-digit whole numbers. <br> NC.4.NBT. 1 <br> NC.4.NBT. 2 <br> NC.4.NBT. 7 <br> Use place value understanding and properties of operations to perform multi-digit arithmetic. <br> NC.4.NBT. 4 <br> NC.4.NBT. 5 <br> NC.4.NBT. 6 | Extend understanding of fractions. <br> NC.4.NF. 1 <br> NC.4.NF. 2 <br> Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. <br> NC.4.NF. 3 <br> Use unit fractions to understand operations of fractions. <br> NC.4.NF. 4 <br> Understand decimal notation for fractions, and compare decimal fractions. <br> NC.4.NF. 6 <br> NC.4.NF. 7 | Solve problems involving measurement. <br> NC.4.MD. 1 <br> NC.4.MD. 2 <br> NC.4.MD. 8 <br> Solve problems involving area and perimeter. <br> NC.4.MD. 3 <br> Represent and interpret data. <br> NC.4.MD. 4 <br> Understand concepts of angle and measure angles. NC.4.MD. 6 | Classify shapes based on lines and angles in twodimensional figures. <br> NC.4.G. 1 <br> NC.4.G. 2 <br> NC.4.G. 3 |


| Practice | Explanation and Example |
| :---: | :---: |
| 1. Make sense of problems and persevere in solving them. | Mathematically proficient students in grade 4 know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Fourth graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers. |
| 2. Reason abstractly and quantitatively. | Mathematically proficient fourth grade students should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts. |
| 3. Construct viable arguments and critique the reasoning of others. | In fourth grade mathematically proficient students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain their thinking and make connections between models and equations. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking. |
| 4. Model with mathematics. | Mathematically proficient fourth grade students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fourth graders should evaluate their results in the context of the situation and reflect on whether the results make sense. |
| 5. Use appropriate tools strategically. | Mathematically proficient fourth grader students consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper or a number line to represent and compare decimals and protractors to measure angles. They use other measurement tools to understand the relative size of units within a system and express measurements given in larger units in terms of smaller units. |
| 6. Attend to precisi | As fourth grader students develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot. |
| 7. Look for and make use of structure. | In fourth grade mathematically proficient students look closely to discover a pattern or structure. For instance, students use properties of operations to explain calculations (partial products model). They relate representations of counting problems such as tree diagrams and arrays to the multiplication principal of counting. They generate number or shape patterns that follow a given rule. |
| 8. Look for and express regularity in repeated reasoning. | Students in fourth grade should notice repetitive actions in computation to make generalizations Students use models to explain calculations and understand how algorithms work. They also use models to examine patterns and generate their own algorithms. For example, students use visual fraction models to write equivalent fractions. |

Return to Standards

## Represent and solve problems involving multiplication and division.

NC.4.OA. 1 Interpret a multiplication equation as a comparison. Multiply or divide to solve word problems involving multiplicative comparisons using models and equations with a symbol for the unknown number. Distinguish multiplicative comparison from additive comparison.

## Clarification

A multiplicative comparison is a situation in which one quantity is multiplied by a specified number to get another quantity (e.g., "a is $n$ times as much as $b$ "). In a multiplicative comparison, the underlying question is what factor would multiply one quantity in order to result in the other. Students should be able to identify and verbalize which quantity is being multiplied and which number tells how many times.

Students should be able to translate comparative situations into equations with a variable and then find the value of the variable. Many opportunities to solve contextual problems and write and identify equations and statements for multiplicative comparison should be provided. Likewise, when given an equation students should be able to create a model and a word problem that matches the equation.

Students are expected to distinguish between additive and multiplicative comparisons. Additive comparisons focus on the difference between two quantities. Multiplicative comparisons focus on one quantity being some number times larger than another.

## For example:

## Additive comparison:

Jane has 8 apples and Sam has 5 apples. How many more apples does Jane have than Sam.

## Multiplicative comparison:

Jane has 8 apples and Sam has 5 times as many apples as Jane. How many apples does Sam have?

In this standard the referent, which is the number of times a quantity is larger than or smaller than another quantity, should be limited to 10 or less. Further, while students multiply a fraction by a whole number in Grade 4 (NC.4.NF.4), multiplicative comparison situations are limited to only whole numbers.

Checking for Understanding
Sally is five years old. Her mom is eight times older. How many years older is
Sally's mom compared to Sally?
Possible responses:
Student A:
First, I need to find the age of Sally's mom.


The difference between the ages of Sally's mom and Sally is $40-5=35$.
Student B:
Sally is 5 . Sally's mom is 8 times older which is $8 \times 5$ or 40 . The difference between their ages is $40-5=35$.

## A book costs $\$ 18$. That is 3 times more than a DVD. How much does a DVD

 cost?Possible response:
Student A:
$18 \div p=3$

| $18 \div p=3$ <br> or $3 \times p=18$ |  |  |  |
| :---: | :---: | :---: | :---: |
| 18   <br> $?$ $?$ $?$ |  |  |  |

Student B:
The book is $\$ 18$ and 3 times more than a DVD.
I know that $6 \times 3=18$ so the DVD is $\$ 6$.

## Represent and solve problems involving multiplication and division

NC.4.OA. 1 Interpret a multiplication equation as a comparison. Multiply or divide to solve word problems involving multiplicative comparisons using models and equations with a symbol for the unknown number. Distinguish multiplicative comparison from additive comparison

## Clarification

Checking for Understanding
Brandi walks for 72 minutes on Saturday. Terrence walks for 8 minutes.
a. Write a question about an additive comparison between the amount of time that Brandi and Terrence walked.
b. Solve the additive comparison question that you wrote using a picture or equation.
c. Write a question about a multiplicative comparison between the amount of time that Brandi and Terrence walked.
d. Solve the multiplicative comparison question that you wrote using a picture or equation.

Possible response:
Additive comparison questions: What is the difference between the amount of time that Brandi walked compared to the time that Terrence walked? How many more minutes did Brandi walk compared to
Terrence? How many fewer minutes did Terrence walk than Brandi?
Multiplicative comparison questions: How many times more minutes did Brandi walk compared to Terrence? How many times fewer minutes did Terrence walk compared to Brandi.

## Represent and solve problems involving multiplication and division.

NC.4.OA. 1 Interpret a multiplication equation as a comparison. Multiply or divide to solve word problems involving multiplicative comparisons using models and equations with a symbol for the unknown number. Distinguish multiplicative comparison from additive comparison.

## Multiplication \& Division Situations

|  | Unknown Product $3 \times 6=?$ | Group Size Unknown (How many in each group?" Division) $3 \times ?=18 \text { and } 18 \div 3=?$ | Number of Groups Unknown ("How many groups?" Division) $? \times 6=18 \text { and } 18 \div 6=?$ |
| :---: | :---: | :---: | :---: |
| Equal Groups | There are 3 bags with 6 plums in each bag. How many plums are there in all? <br> Measurement example. You need 3 lengths of string, each 6 inches long. How much string will you need altogether? <br> (Grade 3) | If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? <br> Measurement example. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be? <br> (Grade 3) | If 18 plums are to be packed 6 to a bag, then how many bags are needed? <br> Measurement example. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have? (Grade 3) |
| Arrays \& Area | There are 3 rows of apples with 6 apples in each row. How many apples are there? <br> Area example. What is the area of a 3 cm by 6 cm rectangle? <br> (Grade 3) | If 18 apples are arranged into 3 equal rows, how many apples will be in each row? <br> Area example. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it? (Grade 3) | If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? <br> Area example. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it? (Grade 3) |
| Compare | Compare- Larger Unknown *New in Grade 4. All three numbers should be whole numbers in Grade 4. <br> A blue hat costs $\$ 6$. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? <br> Measurement example. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long? | Compare- Smaller Unknown *New in Grade 4. All three numbers should be whole numbers in Grade 4. <br> A red hat costs $\$ 18$ and that is 3 times as much as a blue hat costs. How much does a blue hat cost? <br> Measurement example. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first? | Compare- Difference Unknown *New in Grade 4. All three numbers should be whole numbers in Grade 4. <br> A red hat costs $\$ 18$ and a blue hat costs $\$ 6$. How many times as much does the red hat cost compared to the blue hat? <br> Measurement example. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first? |

Return to Standards

North Carolina Department of
NC 4 ${ }^{\text {th }}$ Grade Math Unpacking - Revised June 2022

## Use the four operations with whole numbers to solve problems.

NC.4.OA. 3 Solve two-step word problems involving the four operations with whole numbers.

- Use estimation strategies to assess reasonableness of answers.
- Interpret remainders in word problems.
- Represent problems using equations with a letter standing for the unknown quantity


## Clarification

The focus in this standard is to have students use and discuss various strategies for solving word problems using all four operations. Students should build on the problem solving strategies they developed in earlier grades and apply those strategies to multi-step problems.

Students should be introduced to a variety of estimation strategies. Estimation strategies include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations using various estimation strategies. Estimation strategies include, but are not limited to:

- front-end estimation with adjusting (using the highest place value and estimating from the front end, making adjustments to the estimate by taking into account the remaining amounts),
- rounding and adjusting (students round down or round up and then adjust their estimate depending on how much the rounding affected the original values),
- using friendly or compatible numbers such as factors (students seek to fit numbers together - e.g., rounding to factors and grouping numbers together that have round sums like 100 or 1000),
- using benchmark numbers that are easy to compute (students select close whole numbers for fractions or decimals to determine an estimate).

Problems should be structured so that all acceptable estimation strategies will arrive at a reasonable answer. The assessment of estimation strategies should only have one reasonable answer (500 or 530 ), or a range (between 500 and 550).

In this standard, students are required to solve division tasks and interpret remainders. All problems involving remainders should be in a real-world context that influences how the remainder should be interpreted.

In both Grades 4 and 5 here are ways that students are expected to interpret remainders:

## Checking for Understanding

Two-step word problem with addition and estimation
On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many miles did they travel total? How do you know your answer is reasonable?

## Possible responses:

## Student

I first thought about 267
and 34 . I noticed that
their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500 .

Student 2
I first thought about
194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34 When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with 500 .

## Student 3

I rounded 267 to 300 . I rounded 194 to 200 . I rounded 34 to 30. When I added 300, 200 and 30, I know my answer will be about 530.

Two-step word problem with multiplication and subtraction and estimation: Your class is collecting bottled water for a service project. The goal is to collect 300 bottles of water. On the first day, Max brings in 6 packs with 6 bottles in each container. About how many bottles of water still need to be collected?

Possible responses:

## Student 1

First, I multiplied 6 and 6 which equals 36. I'm trying to get to 300. 36 is close to 40 , and 40 plus 60 is 100. Then I need 2 more hundreds. So, we still need about 260 bottles.

Student 2
First, I multiplied 6 and 6 which equals 36. I know 36 is about 40 and $300-40=260$, so we need about 260 more bottles.

## Use the four operations with whole numbers to solve problems.

NC.4.OA. 3 Solve two-step word problems involving the four operations with whole numbers.

- Use estimation strategies to assess reasonableness of answers.
- Interpret remainders in word problems.
- Represent problems using equations with a letter standing for the unknown quantity.
Clarification

| There are 19 pens that need to be shared between 3 friends and myself <br> $(19$ 4) |  |  |
| :--- | :--- | :--- |
| Give quotient and <br> remainder <br> If we leave the leftover <br> pens on the table how <br> many pens does each <br> person get? (4) <br> How many pens are <br> left on the table? (3) | Put remainder in 1 <br> group <br> lf we give all of the <br> leftovers to only 1 <br> person how many <br> pens will people <br> receive? <br> 3 people receive 4 <br> pens <br> 1 person receives 7 <br> pens. | Share remainder <br> among groups <br> If we give the leftovers to <br> different people until we <br> run out how many pens <br> will people receive? <br> 3 people receive 5 pens <br> 1 person receives 4 pens |
| There are 130 children going on the field trip. Twenty-four children can fit <br> on a bus. |  |  |
| Adding 1 to the <br> quotient <br> How many busses are <br> needed in order to <br> take all of the <br> children? (6) | Give quotient and <br> remainder <br> How many of the <br> busses have 24 <br> children? (5) <br> How many children <br> are on the bus that <br> does not have 24 <br> children? (10) |  |

## Checking for Understanding

Multiplication and division with estimation and a remainder
Ana bakes 13 dozen cookies. She then puts the into bags with 5 cookies in each bag. She keeps the leftover cookies for herself. How many bags of cookies does she have? How many cookies does she keep for herself? Estimate the number of bags before solving.

Possible responses:
Student A:
For my estimate, 13 and 12 are close to 10 so Ana makes about 100 cookies. If she puts them in 5 bags, she will have about 20 bags since $100 \div 5=20$.

In order to solve this problem, first I multiplied 13 and 12.


## ing 1 to the

## quotient

How many busses are needed in order to take all of the children? (6)

## Give quotient and

How many of the busses have 24 children? (5) How many children does not have 24 children? (10)

5 | 1 |
| :---: |
| 10 |
| 20 |
| 156 |
| $-100(20 \times 5)$ |
| 56 |
| $-50(10 \times 5)$ |
| 6 |
| -5 |
| 1 |$(1 \times 5)$

In order to determine how many bags of 5 I can make with my 156 cookies / solved $156 \div 5=$

The answer using partial quotients is $20+10+1$ which is 31. Ana can make 31 bags of cookies. There is a remainder of 1 so Ana can have 1 cookie for herself.

## Use the four operations with whole numbers to solve problems.

NC.4.OA. 3 Solve two-step word problems involving the four operations with whole numbers.

- Use estimation strategies to assess reasonableness of answers.
- Interpret remainders in word problems.
- Represent problems using equations with a letter standing for the unknown quantity.


## Clarification

## Checking for Understanding

## Interpret remainders in word problems

Write different word problems involving $44 \div 6=$ ? where the answers are best represented as:

Problem A: 7
Problem B: 7 r 2
Problem C: 8
Problem D: 7 or 8
Problem E: 9
Possible responses
Problem A: 7. Mary had 44 pencils. Six pencils fit into each of her pencil pouches. How many pouches did she fill? $44 \div 6=p ; p=7 r 2$.
Mary can fill 7 pouches completely.
Problem B: 7 r 2. Mary had 44 pencils. Six pencils fit into each of her pencil pouches. How many pouches could she fill and how many pencils would she have left? $44 \div 6=p ; p=7 r 2$; Mary can fill 7 pouches and have 2 left over.

Problem C: 8. Mary had 44 pencils. Six pencils fit into each of her pencil pouches. What would the fewest number of pouches she would need in order to hold all of her pencils? $44 \div 6=p ; p=7$ r 2; Mary needs 8 pouches to hold all of the pencils.

Problem D: 7 or 8. Mary had 44 pencils. She divided them equally among her friends before giving one of the leftovers to each of her friends. How many pencils could her friends have received? $44 \div 6=p$; $p=7$ r 2; Some of her friends received 7 pencils. Two friends received 8 pencils.

Problem E: 9. Mary had 44 pencils. She put them into 6 different bags. All of the remaining pencils were put into one bag. How many pencils were in the bat that had the most pencils?

There are 156 students going on a roller coaster. If each car of the roller coaster holds 8 students, how many roller coaster cars are needed?
$156 \div 8=b ; b=19 R 4$; They will need 20 cars because 19 cars would not hold all of the students.

## Gain familiarity with factors and multiples.

NC.4.OA. 4 Find all factor pairs for whole numbers up to and including 50 to:

- Recognize that a whole number is a multiple of each of its factors.
- Determine whether a given whole number is a multiple of a given one-digit number.

Determine if the number is prime or composite.

## Clarification

This standard requires students to demonstrate understanding of factors and multiples of whole numbers up to and including 50. Factor pairs include two numbers that when multiplied result in a particular product. Students should be given opportunities to explore factor pairs with concrete objects and drawings to represent arrays.

Multiples are the result of multiplying two whole numbers. Multiples can be related to factors, and this relationship can be discovered through exploration with arrays. Students can build on their understanding of skip counting by a given number to determine the multiples of the given number.

As students explore and discover patterns, they build a conceptual understanding of prime and composite numbers. Prime numbers have exactly two factors, the number one and their own number. For example, the number 17 has the factors of 1 and 17 . Composite numbers have more than two factors. For example, 8 has the factors $1,2,4$, and 8 . A common misconception is that the number 1 is prime, when it is neither prime nor composite. Another common misconception is that all prime numbers are odd numbers. This is not true, since the number 2 has only 2 factors, 1 and 2, and is also an even number.

## Checking for Understanding

Recognize that a whole number is a multiple of each of its factors
Part 1:
There are 24 chairs in the art room. What are the different ways that the chairs can be arranged into equal groups if you want at least 2 groups and want at least 2 chairs in each group?

- How do you know that you have found every arrangement? Write division equations to show your answers. Explain how you know that you have found every arrangement.

Part 2:
There are 48 chairs in the multi-purpose room. What are the different ways that the chairs can be arranged into equal groups if you want at least 2 groups and want at least 2 chairs in each group?

- How do you know that you have found every arrangement? Write division equations to show your answers.
- What relationship do you notice about the size of the groups if the chairs were arranged in 4 groups in both Part 1 and Part 2?
- What about if the chairs were arranged in 8 groups? Explain why you think this relationship exists.

Possible response:
Part 1: 2 groups of 12, 3 groups of 8, 4 groups of 6, 6 groups of 4, 12 groups of 2. I know I have found every group because the number of groups and group sizes should be all of the factors of 24 except for the numbers 1 and 24.

Part 2: 2 groups of 24, 3 groups of 16, 4 groups of 12, 6 groups of 8,8 groups of 6,12 groups of 4, 16 groups of 3, 24 groups of 2.
I noticed that in Part 2 the number of chairs in a group is double or twice as large for the same number of groups. For example, Part 1 had 4 groups of 6 and Part 2 had 4 groups of 12.

In 8 groups we have 8 groups of 3 in Part 1 and in Part 2 we have 8 groups of 6 .

A landscaping company visits the school to talk about the possible ways to tile a patio and picnic area near the playground. The school can afford between 24 and 30 square tiles.

- For each of the proposed number of tiles (24-30), determine all of the possible dimensions of rectangles you could make.
- The space for the patio is configured so that there cannot be any more than 10 tiles in a row. For the proposed number of tiles (24-30), determine which numbers would work as the total number of tiles.
- Which number of tiles provides the most flexibility in terms of the possible ways that the tiles could be arranged? Explain your reasoning.
- Look at the number 29. How many different rectangles can you make? Explain whether 29 is a prime or composite number.


## Gain familiarity with factors and multiples

NC.4.OA. 4 Find all factor pairs for whole numbers up to and including 50 to:

- Recognize that a whole number is a multiple of each of its factors.
- Determine whether a given whole number is a multiple of a given one-digit number.
- Determine if the number is prime or composite.


## Clarification

Checking for Understanding
Determine whether a given whole number is a multiple of a given-one digit number
Katrina and Nico are talking about the number 6 and 2. Katrina says that 6 is a factor of 2, while Nico says that 6 is a multiple of 2. Who is correct? Use multiplication and division equations as well as the words factor and multiple to explain your answer.

## Possible Response:

Student A:
I put the counters into 2 rows with 3 counters in each row. Since 2 and 3 are the dimensions of my array I know that 2 and 3 are factors and 6 is a multiple.


Student B:
Nico is correct. Multiples are the products when you multiply two factors. We know that $6 \div 2=3$ and $2 \times 3$ $=6$ so 6 is a multiple and 2 is a factor of 6 .

## Recognize if a number is prime or composite

Each number below is the area of a rectangle
$11,12,13,14$

1. Use square tiles and make all of the rectangles that you can that have that area.
2. Complete the table below. Based on the number of rectangles that you were able to make state whether each number is prime or composite.

| Area | Dimensions of Rectangles | Prime or Composite |
| :--- | :--- | :--- |
| 11 |  |  |
| 12 |  |  |
| 13 |  |  |
| 14 |  |  |

Based on the number of rectangles that you were able to make state whether each number is prime or composite.

## Gain familiarity with factors and multiples

NC.4.OA. 4 Find all factor pairs for whole numbers up to and including 50 to:

- Recognize that a whole number is a multiple of each of its factors.
- Determine whether a given whole number is a multiple of a given one-digit number
- Determine if the number is prime or composite.


## Clarification

Checking for Understanding
Possible Response:

| Area | Dimensions of Rectangles | Prime or Composite |
| :--- | :--- | :--- |
| 11 | $1 \times 11,11 \times 1$ | Prime |
| 12 | $1 \times 12,2 \times 6,3 \times 4,4 \times 3,6 \times 2,12 \times 1$ | Composite |
| 13 | $1 \times 13,13 \times 1$ | Prime |
| 14 | $1 \times 14,2 \times 7,7 \times 2,14 \times 1$ | Composite |

Vikas says that 37 is a prime number since it ends in a 7 . Is Vikas correct that all numbers that end in a 7 are prime? Look at the numbers that are less than 50 that have a 7 in them. Is each one prime or composite?

Use multiplication or division equations to support your answer
Possible Response:
Student A:
Vikas is not correct. 27 is not prime since $9 \times 3=27$. The other numbers $7,17,37$, and 47 are prime since the only multiplication equation that equals those numbers include the factors 1 and that number

Student B:
Vikas is not correct. 27 has the factors $1,3,9$, and 27. The other numbers $7,17,37$, and 47 are prime since those numbers only have 2 factors which are 1 and itself.

## Generate and analyze patterns.

NC.4.OA.5 Generate and analyze a number or shape pattern that follows a given rule.

## Clarification

In this standard, students must generate or create a number or shape pattern when they are given 1 rule. Students are also expected to analyze a pattern in order to determine or generate the rule that was used to create the pattern.

The standard is a building block for alter grades since the ability to recognize and explain patterns in mathematics leads students to developing the ability to make generalizations, a foundational concept in algebraic thinking. Students need multiple opportunities creating, extending, and analyzing number and shape patterns.

In terms of vocabulary, patterns and rules are related. A pattern is a sequence that repeats the same process over and over. A rule dictates the process.

In Grade 4 students are expected to generate, analyze, and describe patterns that are growing patterns. Growing patterns are generated by either following a repeating rule (add 2 to the previous term) or a rule in which the rule changes each time (add 1 more than we did to generate the previous term).

| Pattern | Example |  |  |  |  | Rule |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Repeating pattern | $\Delta \Delta \square \square$ |  |  |  |  | The core of the pattern is triangle, triangle, square and it repeats. |
| Growing pattern with a repeating rule | Term 1 2 3 4 5 <br> Number 2 4 6 8 10 <br>  +2     <br> +2 $\underbrace{}_{+2}$     <br> +2      |  |  |  |  | The pattern is a growing pattern since the number increases. The rule repeats since the next term is always generated by adding 2 to the previous term. |
| Growing pattern with a changing rule |  |  |  |  |  | The pattern is a growing pattern since the number we add to each term changes. The rule is to add 2 more than we added to create the previous term. |

## Checking for Understanding

Generating a pattern and describing a rule:
Ted and Nancy both mow lawns during the summer to earn money.
Ted charges $\$ 6$ per hour.
Nancy charges $\$ 12$ per hour.
Complete the table to show how much Ted and Nancy would each earn based on the amount of time that it took to mow a lawn.

Based on the data in the table, what is the rule for Ted? What is the rule for Nancy?

Possible response:

|  | Ted | Nancy |
| :---: | :---: | :---: |
| $1 / 2$ hour | 3 | 6 |
| 1 hour | 6 | 12 |
| 1 and $1 / 2$ hours | 9 | 18 |
| 2 hours | 12 | 24 |
| 2 and $1 / 2$ hours | 15 | 30 |
| 3 hours | 18 | 36 |
| 3 and $1 / 2$ hours | 21 | 42 |
| 4 hours | 24 | 48 |

There are 4 beans in the jar. Each day 3 beans are added. How many beans are in the jar for each of the first 5 days? Describe the rule for the pattern.

Possible response:

| Day | Beans |
| :---: | :---: |
| 0 | 4 |
| 1 | 7 |
| 2 | 10 |
| 3 | 13 |
| 4 | 16 |
| 5 | 19 |

The rule is that 3 more beans are added each day, which means we add 3 to the previous term in the pattern.

## Checking for Understanding

A banquet company provides options for table arrangements: triangular tables, square tables, and hexagonal tables. For each type of table, you can fit one person on each side of the table. For parties, they want to put all of the tables together so that every table shares at least one side with another table.

Based on this proposed arrangement, how many people could you sit at 1 triangular table? 2 connected triangular tables? 3 connected triangular tables? 4 connected triangular tables?

Solutions:
1 table: 3 people
2 tables: 4 people
3 tables: 5 people
4 tables: 6 people

Analyzing a pattern and describing the rule
Unique decides to walk each day after school with her friend. Below is the number of miles she walks each day.


Based on the information in the table is this a growing pattern or a repeating pattern? What is the rule for the number of miles that Unique walks?

How many miles will Unique walk on Day 6 ? How many miles will Unique walk on Day 7? Explain how you know.

Possible response:
The pattern is a growing pattern since the number we add to the number before increases or grows.
The rule is that we find the next number by adding 1 more than we did to find the previous number.

## Generate and analyze patterns.

NC.4.OA.5 Generate and analyze a number or shape pattern that follows a given rule.

Checking for Understanding
Nina looks at the pattern below.


Complete the table below based on the pattern.
How many squares will be in the 5th, 6th, and 7th terms?
What is the rule that is used to generate the pattern? Explain how you know.
Possible Response:

| Term | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Number | 2 | 6 | 12 | 20 | 30 | 42 | 56 |
| Difference between number <br> and previous number | +2 | +4 | +6 | +8 | +10 | +12 | +14 |

The rule that was used to generate the pattern was to add one row and one column to the previous picture. By adding a row and a column you are adding 2 more squares than the previous term. Specifically, the fifth term has 30 squares which is 10 more than the term before. The pattern was generated by adding 2, then adding 4, then adding 6 and so on.


Return to Standards

## Number and Operations in Base Ten

## Generalize place value understanding for multi-digit whole numbers.

NC.4.NBT. 1 Explain that in a multi-digit whole number, a digit in one place represents 10 times as much as it represents in the place to its right, up to 100,000 .

## Clarification

## Checking for Understanding

This standard calls for students to extend their understanding of place value by exploring and explaining the relationship between the magnitude of a digit and the value of that same digit if it were one place to the left of the original digit.

Students are expected to reason about the magnitude of digits in a number and make connections to the idea that a digit to the left of a number has a value that is 10 times greater than the value of a digit to the immediate right.

What is the value of the 3 in hundreds place compared to the value of the 3 in the ones place in the number 353 ? Use pictures, equations, or words to support your answer.

Possible responses.
Student A:
$353=300+50+3$. The 3 in the hundreds place has a value of 300 . The 3 in the ones place has a value of 3 . The 3 in the hundreds place is 100 times greater than the 3 in the ones place.

Student B:
With base ten blocks I see that the 3 in the hundreds place is 300 and the 3 in the ones place is 3 . There are 100 ones in every group of 100 so 300 is 100 times more than the 3 in 353.


䧄:


Student C:
$3 \times 10$ is 30 and $3 \times 10 \times 10$ is 300 . So the value of the 3 in the hundreds place is $10 \times 10$ or 100 times greater than the 3 in the ones place.

Brandi said, "In my pocket I have 25 of the same amount of dollar bills.
Part 1: For each of the scenarios below, write an equation and determine the value of Brandi's money.
a) 25 one dollar bills
b) 25 ten dollar bills
c) 25 hundred dollar bills
d) 25 one thousand dollar bills

Part 2: Brandi is trying to determine the relationship between the value of the 5 in 250 and the value of the 5 in 25,000. Use pictures, equations, or words to explain the relationship between the 5 's in the two numbers.

## Generalize place value understanding for multi-digit whole numbers.

NC.4.NBT. 1 Explain that in a multi-digit whole number, a digit in one place represents 10 times as much as it represents in the place to its right, up to 100,000

| Clarification | Checking for Understanding |
| :---: | :---: |
|  | Possible Answers: <br> Part 1: $25 \times 1=25 ; 25 \times 10=250 ; 25 \times 100=2,500 ; 25 \times 1,000=$ 25,000 <br> Part 2: In 250 the 5 has a value of 50. In 25,000 the 5 has a value of 5,000 . In order to move the 5 one place to the left we have to multiply by 10 so $50 \times 10=500$ $50 \times 10 \times 10=5,000$ <br> That means that the 5 in 250 is $10 \times 10$ or 100 times less than the 5 in 25,000. |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

## Generalize place value understanding for multi-digit whole numbers.

NC.4.NBT. 2 Read and write multi-digit whole numbers up to and including 100,000 using numerals, number names, and expanded form.

## Clarification

This standard asks for students to write numbers in various forms. Students should have flexibility with the different forms of a number, including numbers that are in nontraditional forms, where there is a number greater than 9 in a given place.

The written form or number name of a number requires students to write out a number in words such as $285=$ two hundred eighty-five. Traditional expanded form is $285=200+80+5$. However, students should have opportunities to explore the idea that 285 in nontraditional forms could also be written as 28 tens plus 5 ones or 1 hundred, 18 tens, and 5 ones. They should also be comfortable with expanding a number by place value using multiplication in the notation, such as $285=(2 \times 100)$ $+(8 \times 10)+(5 \times 1)$.

In this standard, students need to understand the role of commas. Each group of 3 digits between commas is read as hundreds, tens, and ones followed by the appropriate unit. For example, the number 97,345 would be read ninety-seven thousand, three hundred forty-five.

## Checking for Understanding

Place value with numerals and expanded form
Juice pouches are packaged in different ways. A box holds 10 pouches. A case holds 10 boxes. A crate holds 10 cases.

Some students bring in juice boxes for Field Day. The information is below
Miguel- 1 crate, 12 cases, 3 boxes and 6 pouches.
Aaron- 1 crate, 13 cases, 17 boxes, and 2 pouches.
Sarah- 1 crate, 12 cases, 2 boxes and 17 pouches.
Vicky- 1 crate, 14 cases, 6 boxes, and 9 pouches.

1) If each person were going to reorganize their drink pouches to use as many of the larger containers as possible, how many of each container would each of them need?
2) How many total drink pouches does each student have?

Place value with number names and expanded form (integrated comparisons NC.4.NBT.7)
Which of the following is greater than 4,050 ? Use a place value chart or equations to support your answer.
A. thirty-nine hundreds, 14 tens, 12 ones
B. thirty-eight hundreds, 24 tens, 9 ones
C. two hundred more than thirty-seven hundreds, 14 tens, 8 ones
D. forty hundreds, 4 tens, 19 ones

Possible answers

|  | $\underline{\text { Th }}$ | $\underline{H u}$ | Tens | Ones | Number |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $A$ | 4 | $\begin{aligned} & 39+1 \\ & 40=4 \mathrm{Th}+0 \mathrm{H} \\ & 0 \end{aligned}$ | $\begin{aligned} & 14+1 \\ & 15=1 H+5 T \\ & 5 \end{aligned}$ | $\begin{aligned} & 12=1 T+20 \\ & 2 \end{aligned}$ | 4,052 |
| $B$ | 4 | $\begin{aligned} & 38+2 H \\ & 40=4 T h+0 H \\ & 0 \end{aligned}$ | $24=2 H+4 T$ | 9 | 4,049 |
| C | $\begin{aligned} & 3+1 \\ & 4 \end{aligned}$ | $\begin{aligned} & 37+1 \\ & 38=3 T h+8 H \\ & 8+2 H=10 \\ & 10=1 T h+0 H \\ & 0 \end{aligned}$ | $14=1 \mathrm{H}+4 T$ | 8 | 4,048 |
| D | 4 | $\begin{aligned} & 40=4 t h+0 H \\ & 0 \end{aligned}$ | $\begin{aligned} & 4+1 \\ & 5 \end{aligned}$ | $\begin{aligned} & 19=1 T+90 \\ & 9 \end{aligned}$ | 4,059 |

## Generalize place value understanding for multi-digit whole numbers.

NC.4.NBT. 7 Compare two multi-digit numbers up to and including 100,000 based on the values of the digits in each place, using >, =, and < symbols to record the results of comparisons.

## Clarification

In this standard, students use their understanding of groups and value of digits to compare two numbers by examining the value of the digits. Students are expected to be able to compare numbers presented in various forms, including nontraditional forms here the value is greater than 9 for a given place.

Students should have ample experiences communicating their comparisons in words before using symbols. Students were introduced to the symbols greater than ( $>$ ), less than ( $<$ ) and equal to ( $=$ ) in Grade 1 and continued to use them in Grade 2 to compare whole numbers, and in Grade 3 to compare fractions.

While students may have the skills to order more than 2 numbers, this standard focuses on comparing two numbers and using reasoning about place value to support the use of the various symbols. This standard may be assessed by having students order up to 4 numbers based on the understanding that ordering those numbers includes comparing two numbers at a time.

## Checking for Understanding

Compare these two numbers. 75,452 $\qquad$ 75,455

Possible responses:
Student A
Place Value
75,452 has 75 thousands, 4 hundreds, 5 tens, and 2 ones. 75,455 has 75
thousands, 4 hundreds, 5 tens, and 5 ones. They have the same number of thousands, hundreds and the same number of tens, but 455 has 5 ones and 75,452 only has 2 ones. 75,452 is less than 455.

$$
75,452<75,455
$$

Find the population of these cities in number form. Then put them in order from least to greatest:

Thomasville- 17 thousands, 98 tens, 14 ones
Henderson- 17 thousands, 2 hundreds, 15 ones
Elizabeth City- 17 thousands, thirty-two tens, five ones
Davidson- 1 ten thousand, 34 hundreds, 27 ones
Possible answer:

|  | Ten Th | Th | $H$ | $T$ | $O$ | Number |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Thom |  | 17 |  | 98 <br> 99 <br> 1 | 14 <br> 9 |  |
|  | 1 | 7 | 9 | 9 | 17,994 |  |
| Hend | 1 | 17 | 2 |  | 15 | 17,215 |
|  | 1 | 7 |  | 1 | 5 |  |
| Eliz | 1 | 17 <br> 7 | 3 | 32 <br> 2 | 5 | 17,325 |
| Dav | 1 | 1 | 3 | 34 <br> 4 | 2 | 27 <br> 7 |

## Student B Counting

75,452 is less than 75,455 . I know this because they have the same thousands. So, I'm going to compare 452 and 455. When I count up I say 452 before I say $455.75,452$ is less than 75,455

$$
75,452<75,455
$$

Possible answer:

Deavidson: 13,427
Davidson: 13,427 Henderson: 17,215 Elizabeth City: 17,325 Thomasville: 17,994

## Use place value understanding and properties of operations to perform multi-digit arithmetic.

## NC.4.NBT. 4 Add and subtract multi-digit whole numbers up to and including 100,000 using the standard algorithm with place value understanding

## Clarification

In this standard, students build on their conceptual understanding of addition and subtraction, their use of place value and their flexibility with multiple strategies to make sense of the standard algorithm. They continue to use place value in describing and justifying the processes they use to add and subtract. Students are expected to explain their thinking to show understanding of the algorithm

This is the first grade level in which students are expected to be proficient at using the standard algorithm to add and subtract. However, other previously learned strategies are still appropriate for students to use prior to exploring and developing proficiency with the algorithm. In Grade 3 students use expanded form and drawings of base 10 blocks to solve addition and subtraction problems.

In mathematics, an algorithm is defined by its steps and not by the way those steps are recorded in writing. With this in mind, minor variations in methods of recording standard algorithms are acceptable.

Students may ask if it is possible to subtract a larger number from a smaller number. While it is not the focus or expectation of this standard in this grade students should know that it is mathematically possible, and they will be learning more about that concept in later grades. If the misconception that larger numbers cannot be subtracted from smaller numbers is confirmed or reinforced, students may struggle to make the transition to negative numbers in later grades.

## Checking for Understanding

The following amounts of juice were in separate containers after the school's parent breakfast.

- Container 1: 750 mL
- Container 2: $1,450 \mathrm{~mL}$
- Container 3: 2,087 mL

Part 1: If all of the liquid was put into one large container how much liquid would be in the large container?

Part 2: If the container holds $5,000 \mathrm{~mL}$ how much more liquid can still be added until the container is full?

Possible Answer:

| Part 1: | Part 2: |
| :--- | :--- |
|  | 99 |
| 11 | 4101010 |
| 1,450 | $5,-0-0-0$ |
| 2,087 | $\frac{-4,2}{} \quad 8 \quad 7$ |
| $\frac{7750}{4,287}$ | 13. |

On a field trip, three different schools send their fourth graders across town to the high school for a math competition. Each school sends between 120 and 170 students each. There are 417 students total.

1. How many students could have come from each school? Show your thinking
2. Find another possible solution to this task. Show your thinking.

## Possible answer:

The three schools send a total of 417 so the three addends must tota 417. Each should be greater than 120 and smaller than 170 Examples: $140+140+137=417 ; 120+127+170=417$.

## Use place value understanding and properties of operations to perform multi-digit arithmetic.

NC.4.NBT. 5 Multiply a whole number of up to three digits by a one-digit whole number, and multiply up to two two-digit numbers with place value understanding using area models, partial products, and the properties of operations. Use models to make connections and develop the algorithm.

## Clarification

In this standard, students extend their understanding of multiplying a singledigit factor times a multiple of ten (NC.3.NBT.3) to multiplying a single-digit factor times two- or three-digit factors and two two-digit factors.

Students are expected to apply their understanding of place value and various forms of a number to compute products. Students will also use area models, partial products and properties of operations to solve multiplication problems. Parentheses are not expected until grade 5, so students should record multiplication using partial products without parentheses.

Connections should be made between models and written equations (as shown below), but it is not necessary for fourth grade students to use the standard algorithm. The standard algorithm for multiplication is not an expectation until fifth grade.


Fig 18. Methods for multi-digit multiplication using $68 \times 46$ Adapted fromFuson (2003, p. 303)

Checking for Understanding
There are 25 dozen cookies in the bakery. What is the total number of cookies at the bakery?

Possible responses:

| Student A |  |  |
| :--- | :--- | :--- |
| $25 \times 12$ |  |  |
| I broke 12 up into $10+$ | Student B <br> $25 \times 12$ <br> I broke 25 up into 5 <br> groups of 5 | Student C <br> $25 \times 12$ <br> I doubled 25 and cut 12 <br> in half to get $50 \times 6$ |
| $25 \times 10=250$ |  |  |
| $25 \times 2=50$ |  |  |
| $250+50=300$ | $5 \times 12=60$ <br> There are 5 groups of 5 <br> in 25 <br> $60 \times 5=300$ | $50 \times 6=300$ |

In the cafeteria, there are 14 long tables. Each table seats 16 students. How many students can eat in the cafeteria at one time?

Possible responses:
Student A:
Using base ten blocks to model this problem, I broke $14 \times 16$ into this equation:

| 1 | 10 | 100 |
| ---: | :---: | ---: |
| hundred | $x$ | 40 |
| 4 tens | 10 | 60 |
| 6 tens | $4 x$ | 24 |
| 24 ones | 10 |  |
|  | $6 x$ |  |
|  | 10 |  |
|  | $6 x$ |  |
|  | 4 |  |


$14 \times 16=224$

## Use place value understanding and properties of operations to perform multi-digit arithmetic.

NC.4.NBT. 5 Multiply a whole number of up to three digits by a one-digit whole number, and multiply up to two two-digit numbers with place value understanding using area models, partial products, and the properties of operations. Use models to make connections and develop the algorithm.


Return to Standards

## Use place value understanding and properties of operations to perform multi-digit arithmetic.

NC.4.NBT. 6 Find whole-number quotients and remainders with up to three-digit dividends and one-digit divisors with place value understanding using rectangular arrays, area models, repeated subtraction, partial quotients, properties of operations, and/or the relationship between multiplication and division

## Clarification

In this standard, students build on their understanding of the meaning of division and the relationship to multiplication by solving division problems in and out of context that have a three-digit dividend and a one-digit divisor.

The focus of this standard is to build conceptual understanding of division. Students are expected to use various strategies and explain their thinking. Students are not expected to master the traditional algorithm until middle school.

This standard calls for students to explore division through various strategies. Students should be able to apply their understanding of place value and various forms of a number to compute quotients. Students will also use arrays and area models, repeated subtraction, partial quotients and properties of operations to solve division problems

This standard also intersects division situations that have remainders. Refer to NC.4.OA. 3 for examples and information about how students are expected to interpret and make sense of remainders in division situations.

The focus of this standard is to build conceptual understanding of division. Students are expected to use various strategies and explain their thinking. Students are not expected to master the traditional algorithm until middle school.

Checking for Understanding
A 4th grade teacher bought 4 new pencil boxes. She has 260 pencils. She wants to put the pencils in the boxes so that each box has the same number of pencils. How many pencils will there be in each box?

Possible responses:

- Using Base 10 Blocks: Students build 260 with base 10 blocks and distribute them into 4 equal groups. Some students may need to trade the 2 hundreds for tens but others may easily recognize that 200 divided by 4 is 50
- Using Place Value: $260 \div 4=(200 \div 4)+(60 \div 4)=50+15=65$
- Using Multiplication: $4 \times 50=200,4 \times 10=40,4 \times 5=20 ; 50+10+5$ = 65; so, $260 \div 4=65$

There are 592 students participating in Field Day. They are put into teams of 8 for the competition. How many teams get created?


## Use place value understanding and properties of operations to perform multi-digit arithmetic.

NC.4.NBT. 6 Find whole-number quotients and remainders with up to three-digit dividends and one-digit divisors with place value understanding using rectangular arrays, area models, repeated subtraction, partial quotients, properties of operations, and/or the relationship between multiplication and division Clarification Checking for Understanding
Journey has 150 hair bows. She puts them into bags with 6 hair bows in each bag. How many bags of hair bows will she have?

Possible Responses:
Open Array/Area Model


$$
\begin{aligned}
150-60 & =90 \\
90-60 & =30 \\
30-30 & =0
\end{aligned}
$$

I started thinking about how many groups of 6 are in 250. I knew that $10 \times 6=60$ so I did that 2 times which meant that I had 20 groups of 6 which is 120 . I knew that 120 is 30 from 150 and I knew that $5 \times 6=30$ so I could make 5 more bags of hair bows with the 30 that is left. My total number of bags is $10+10+5$ which is 25 .

Partial Quotients

6 \begin{tabular}{rr}
5 \& 20 <br>
20 \& <br>

| 150 |
| :--- |
| $\frac{-120}{30}$ |
| $\frac{-30}{0}$ |$(20 \times 6)$

\end{tabular}

## Number and Operations-Fractions

## Extend understanding of fractions.

NC.4.NF. 1 Explain why a fraction is equivalent to another fraction by using area and length fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size.

## Clarification

In this standard, students are expected to use area and length fraction models to explain how fractions are equivalent to each other. Area models include circles and rectangles while length models typically focus on number lines. Students should not do any work on this standard without the use of a model. Students only work with the denominators $2,3,4,5,6,8,10,12$, and 100 in this standard.

## Checking for Understanding

Kennedy rode her bike down a straight road and stopped at the halfway point for water. Courtney also biked the same distance but broke her bike ride into 4 equal parts to get water. Did Courtney and Kennedy ever stop at the same place to get water? How do you know? Draw and label a number line to support your conclusions.

Possible student response:
Courtney broke her ride into fourths since she had 4 equal parts.
Kennedy stopped at the halfway point. Based on the number line
Courtney stopped $\frac{2}{4}$ of the way down the road which is the same point as one half.


Mr. Gomez and Mr. Lopez each have vegetable gardens that are the same size. Mr. Gomez plants carrots in 6/8 of his garden. If Mr. Lopez has 4 regions and wants to plant carrots in the same sized space as Mr. Gomez how many of the regions will he plant carrots in? Draw a picture and write a sentence to explain your answer.

Possible response:
 Mr. Lopez:
I know that $\frac{2}{8}=\frac{1}{4}$ and $\frac{6}{8}=\frac{2}{8}+\frac{2}{8}+\frac{2}{8}$. That means that Mr. Lopez will have carrots in $\frac{3}{4}$ of his garden since $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=\frac{3}{4}$.

## Extend understanding of fractions.

NC.4.NF. 1 Explain why a fraction is equivalent to another fraction by using area and length fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size.

| Clarification | Checking for Understanding |
| :--- | :--- |
|  | Lauren is trying to think about fractions are equivalent to $\frac{1}{2}$. |
|  | Part 1: Using the denominators $4,6,8$, and 12 use models to show all of the |
| fractions that are equivalent to $\frac{1}{2}$. |  |
|  | Part 2: Pick one of the fractions that is equivalent to $\frac{1}{2}$. Explain how you know |
| that fraction is equivalent to $\frac{1}{2}$. |  |
|  | Possible Response: |
| Part 1: Number lines or area models for any of the following fractions. |  |
| Part 2: The explanation clearly refers to a model and explains why the |  |
| fraction is equivalent to $\frac{1}{2}$. |  |

## Extend understanding of fractions.

NC.4.NF. 2 Compare two fractions with different numerators and different denominators, using the denominators $2,3,4,5,6,8,10,12$, and 100. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>,=$, or $<$, and justify the conclusions by:

- Reasoning about their size and using area and length models.
- Using benchmark fractions $0,1 / 2$, and a whole.
- Comparing common numerator or common denominators.


## Clarification

In this standard, students are expected to compare two fractions with the denominators $2,3,4,5,6,8,10,12$, and 100.

In Grade 3 students used reasoning and models to compare fractions that either had the same numerator or the same denominator. In Grade 4, students reason about their size and justify their comparison using area and length models, including circles, rectangles, and number lines. Students are also expected to use the benchmark fractions $0,1 / 2$ and 1 whole to compare fractions.

Students should be able to put a set of up to 4 fractions in order based on their size by comparing pairs of fractions.

Students should not use a procedure such as cross multiplication, for comparing fractions. A student's justification that relies solely on explaining the steps of an algorithm would not demonstrate mastery of this standard.

## Checking for Understanding

Crystal and Katie are each running a mile. Crystal ran $\frac{3}{4}$ of a mile before stopping for water, while Katie ran $\frac{2}{3}$ of a mile before stopping. Who ran the farthest before stopping? Draw a picture or write a sentence to support your answer.

Possible responses:


Student 2: Comparing common numerators or denominators
I noticed that Crystal ran $\frac{1}{4}$ less than a whole and Katie ran $\frac{1}{3}$ less than a whole. Since $\frac{1}{4}$ is smaller than $\frac{1}{3}$ I know Crystal ran the farthest.

Tammy, Joe, and Lisa went to the movies. Each of them bought a small box of popcorn. Tammy ate $\frac{3}{6}$ of her popcorn. Joe ate $\frac{3}{8}$ of his popcorn and Lisa ate $\frac{2}{3}$ of her popcorn. Who ate more?

Possible responses:

- I can compare $\frac{3}{6}$ and $\frac{3}{8}$ and I know that sixths are larger than eighths, so $\frac{3}{6}>\frac{3}{8}$. (comparing common numerator or denominators)
- When I compare $\frac{2}{3}$ to $\frac{3}{6}$, I know that $\frac{2}{3}$ is equivalent to $4 / 6$, so $\frac{2}{3}>\frac{3}{6}$. (reasoning about their size)
- I know that $\frac{3}{8}$ is less than half and $\frac{2}{3}$ is more than half so $\frac{3}{8}<\frac{2}{3}$. (using benchmark fractions)
- If I put the fractions in order from least to greatest based on my comparisons: $\frac{3}{8}, \frac{3}{6}, \frac{2}{3}$ Which of the following fractions is smaller than or equal to $\frac{3}{4}$ ? For each fraction explain your reasoning

$$
\frac{2}{3}, \frac{5}{8}, \frac{9}{12}, \frac{5}{6}, \frac{4}{5}
$$

## Extend understanding of fractions.

NC.4.NF. 2 Compare two fractions with different numerators and different denominators, using the denominators $2,3,4,5,6,8,10,12$, and 100. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>,=$, or $<$, and justify the conclusions by:

- Reasoning about their size and using area and length models.
- Using benchmark fractions $0,1 / 2$, and a whole
- Comparing common numerator or common denominators.


Return to Standards

## Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

NC.4.NF. 3 Understand and justify decompositions of fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100.

- Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
- Decompose a fraction into a sum of unit fractions and a sum of fractions with the same denominator in more than one way using area models, length models, and equations.
- Add and subtract fractions, including mixed numbers with like denominators, by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
- Solve word problems involving addition and subtraction of fractions, including mixed numbers by writing equations from a visual representation of the problem.

Clarification
NC.4.NF. 3 calls for students to explore the meaning of addition and subtraction with fractional amounts using both areas and length models. This work is limited to joining, separating, and comparing fractions with like denominators, and the only denominators that should be used are $2,3,4,5,6$, $8,10,12$, and 100.

The second bullet focuses on using area and length models to decompose a fraction or mixed number into smaller fractional amounts, including unit fractions. A unit fraction is a term that identifies the size of 1 fractional piece in a whole and has a 1 in the numerator. For example, $\frac{1}{3}$ is the unit fraction that identifies a whole being divided into 3 equal pieces. Just as there are 3, oneinch units in the length of 3 inches, there are $2, \frac{1}{3}$ units in the fraction $\frac{2}{3}$.

Models should also be used to support students' work when they add and subtract fractions in the latter two bullets of this standard. The work decomposing fractions, including mixed numbers, in the first two bullets of this standard, is foundational for the last two bullets of this Standard.

While equivalent fractions are not explicitly mentioned in this standard, students are expected to identify the equivalent fractions of answers to addition and subtraction problems.

## Checking for Understanding

Decompose a fraction into a sum of unit fractions and a sum of fractions
After a pizza party there is $\frac{7}{8}$ of a pizza left. Some pieces of the pizza are cheese, some are pepperoni, and some are vegetable. What are the possible fractions that could be used to represent the amount of pizza that could be cheese, pepperoni, and vegetable? Draw a picture and write an equation to represent the amounts of each type of pizza that are remaining. Find at least wo combinations.

Possible response


Add and subtraction fractions using the properties of operations and the relationship between addition and subtraction.

The picture shows the amount of crackers that Mrs. Nickel has. She has $\frac{7}{8}$ more of a pack of crackers than Mrs. Fazio. If they combine their crackers how many packs of crackers do they have?


## Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers

NC.4.NF. 3 Understand and justify decompositions of fractions with denominators of $2,3,4,5,6,8,10,12$, and 100.

- Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
- Decompose a fraction into a sum of unit fractions and a sum of fractions with the same denominator in more than one way using area models, length models, and equations.
- Add and subtract fractions, including mixed numbers with like denominators, by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
- Solve word problems involving addition and subtraction of fractions, including mixed numbers by writing equations from a visual representation of the problem.

Clarification

## Checking for Understanding

Possible Response:
Mrs. Nickel has $2 \frac{5}{8}$ packs. I made a part-part-whole picture to help me.
Since she has more than Mrs. Fazio, we need to subtract the difference to find Mrs. Fazio.

$$
2 \frac{5}{8}=\text { Fazio }+\frac{7}{8} \text { or Fazio }=2 \frac{5}{8}-\frac{7}{8}
$$



Mrs. Nickel $2 \frac{5}{8}$

I crossed out the $\frac{5}{8}$ then I partitioned one
of the other wholes into eighths then crossed out $\frac{2}{8}$ from that whole since I had to cross out a total of $\frac{7}{8}$. Mrs. Fazio had $1 \frac{6}{8}$. The total is $1 \frac{6}{8}+2 \frac{5}{8}$. I $\square$

solved that by adding $1+2+\frac{6}{8}+\frac{5}{8}=3 \frac{11}{8}$.
I know that $\frac{11}{8}$ is $1 \frac{3}{8}$ so that means my total is $3+1+\frac{3}{8}=4 \frac{3}{8}$.
Which of the following fractions make this statement true?

a) $\frac{11}{8}$, b) $1 \frac{1}{6}$, c) $1 \frac{1}{4}$, d) $1 \frac{1}{3}$, f) $1 \frac{2}{10}$

Possible response:
Since addition and subtraction are opposites, I need to find fractions greater than $\frac{5}{8}+\frac{5}{8}$ which is $\frac{10}{8}$ or $1 \frac{2}{8}$.
*Comparisons can be made using length or area models, or reasoning about the size of each fraction.
a) yes, b) no, c) no, d) yes, f) no

## Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

NC.4.NF. 3 Understand and justify decompositions of fractions with denominators of $2,3,4,5,6,8,10,12$, and 100.

- Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
- Decompose a fraction into a sum of unit fractions and a sum of fractions with the same denominator in more than one way using area models, length models, and equations.
- Add and subtract fractions, including mixed numbers with like denominators, by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
- Solve word problems involving addition and subtraction of fractions, including mixed numbers by writing equations from a visual representation of the problem.

Clarification

## Checking for Understanding

## Solve word problems involving addition and subtraction of fractions

There is some firewood on the pile. Mr. Mickelson adds $\frac{7}{8}$ pounds of firewood. If there is now $2 \frac{1}{8}$ of firewood on the pile how much firewood was first there?

Possible student responses:

Student 1
I wrote the equation $2 \frac{1}{8}-\frac{7}{8}$ to find out how much firewood was first there. I then drew a picture of $2 \frac{1}{8}$ and crossed out
$\frac{7}{8}$. I had $1 \frac{2}{8}$ or $1 \frac{1}{4}$ left.


Student 2
I drew $\frac{7}{8}$ and then added on until I reached $2 \frac{1}{8}$. I then went back and counted. I added $\frac{1}{8}+1+\frac{1}{8}$ which is $1 \frac{2}{8}$ or $1 \frac{1}{4}$ pounds.


## Student 3

I renamed $2 \frac{1}{8}$ into an equivalent fraction $\frac{17}{8}$. I then took $\frac{7}{8}$ away from $\frac{17}{8}$ which got me an answer of $\frac{10}{8}$. When I drew the picture, I realized $\frac{10}{8}$ is 1 whole and $\frac{2}{8}$, which is $1 \frac{2}{8}$ pounds.

## Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

NC.4.NF. 3 Understand and justify decompositions of fractions with denominators of $2,3,4,5,6,8,10,12$, and 100.

- Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
- Decompose a fraction into a sum of unit fractions and a sum of fractions with the same denominator in more than one way using area models, length models, and equations.
- Add and subtract fractions, including mixed numbers with like denominators, by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
- Solve word problems involving addition and subtraction of fractions, including mixed numbers by writing equations from a visual representation of the problem.



## Checking for Understanding

Brielle ran $1 \frac{2}{3}$ miles less than Kim. Brielle ran $2 \frac{2}{3}$ miles. How far did Kim run? Draw a number line and an equation to support your answer.

Possible student responses:
Student 1:
I started at $2 \frac{2}{3}$ since that was how far Brielle ran. Since Brielle ran less
than Kim I knew I had to add $1 \frac{2}{3}$. I broke the $\frac{2}{3}$ up into 2 jumps of a $\frac{1}{3}$ so I
could land on 3 , then jump to 4 , then landed on 4 and $\frac{1}{3}$. An equation is 2
$\frac{2}{3}+1 \frac{2}{3}=4 \frac{1}{3}$.


Student 2:
I started at $2 \frac{2}{3}$ since that was how far Brielle ran. Since Brielle ran less than Kim I knew I had to add $1 \frac{2}{3}$. I jumped 1 to land on $3 \frac{2}{3}$. I then made 2 jumps of $\frac{1}{3}$ and landed on $4 \frac{1}{3}$. An equation is $2 \frac{2}{3}+1 \frac{2}{3}=4 \frac{1}{3}$.


## Use unit fractions to understand operations of fractions.

NC.4.NF. 4 Apply and extend previous understandings of multiplication to:

- Model and explain how fractions can be represented by multiplying a whole number by a unit fraction, using this understanding to multiply a whole number by any fraction less than one.
- Solve word problems involving multiplication of a fraction by a whole number


## Clarification

This standard calls for students to understand a fraction as a whole number of groups of a unit fraction (e.g. $\frac{3}{8}$ is 3 groups of $\frac{1}{8}$ ). A unit fraction is a term that identifies the size of 1 fractional piece in a whole. For example, $\frac{1}{3}$ is the unit fraction that identifies a whole being divided into 3 equal pieces. Just as there are 3 , one inch units in the length of 3 inches, there are $2, \frac{1}{3}$ units in the fraction $\frac{2}{3}$.

Students also use multiplication of a fraction by a whole number to determine a fraction of a set.

## For example:

There are 12 pizzas. $\frac{1}{4}$ of the total number of pizzas is 3 .


There are 12 people and each person takes $\frac{1}{4}$ of a pizza. The total number of pizzas eaten by 12 people is 3 .


Students use a unit fraction as well as repeated addition to establish a
foundation for the process of multiplying a whole number by a fraction ( $4 \times \frac{2}{3}=$ $\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}=\frac{(4 x 2)}{3}$.

Students use both area and length models to explore and solve word problems. All fractions are limited to the denominators of $2,3,4,5,6,8,10$, 12.

## Checking for Understanding

Express the fraction $\frac{3}{6}$ as the product of a whole number and a unit fraction.
Draw a model which supports your answer.
Possible response:

$$
\frac{3}{6}=3 \times \frac{1}{6} .
$$



Tomas and Hector are running at P.E. Tomas runs $\frac{3}{4}$. of a mile. Hector runs 3 times as far as Tomas. How far did Hector run?

Possible response:


$$
\text { Hector ran } \frac{3}{4} \text { times } 3=\frac{9}{4} \text { miles. }
$$

Michelle was making bracelets from ribbon. She wanted to make 4 bracelets and each bracelet needed $\frac{2}{3}$ yards of ribbon. How much ribbon does Michelle need?

Possible response:
Michelle needs $\frac{8}{3}$ yards of ribbon.


## Use unit fractions to understand operations of fractions.

NC.4.NF. 4 Apply and extend previous understandings of multiplication to:

- Model and explain how fractions can be represented by multiplying a whole number by a unit fraction, using this understanding to multiply a whole number by any fraction less than one.
- Solve word problems involving multiplication of a fraction by a whole number


## Clarification

This standard also includes situations involving finding the fractional amount of a set of objects. In this standard the number in a set is limited to whole numbers within 20. This work should use set models, arrays, or reasoning about fractions as strategies to explore fractions of a set situations.

## Checking for Understanding

Fractions of a Set
There are 20 apples in the basket. $\frac{1}{5}$ of them are red, $\frac{3}{5}$ of them are yellow, and the rest are green. How many apples are there of each color?

Possible response:
Student A:
I drew the 20 apples in the shape of an array with 5 in each row. I then thought about the red apples. Since $\frac{1}{5}$ means 1 out of 5 I shaded the first circle red in each row so there were 4 red. I then shaded the next 3 in each row yellow since $\frac{3}{5}$ means 3 out of 5 . There were 15 yellow. I shaded the rest green which
 were 5 circles.

## Student B:

I started with $\frac{1}{5}$ since it is a unit fraction. I know that $5 \times 4=20$ and $20 \div$ $5=4$, so $\frac{1}{5}$ of 20 is 4 . I then found the yellow apples. Since $\frac{1}{5}$ of $20=4$ then $\frac{3}{5}$ of 20 is going to be $3 \times \frac{1}{5}$ of 20 or $3 \times 4$ which is 12 . Then to find the green I subtracted the red and yellow from 20 so 20-4-12 = 4. There were 4 green apples.

While trying to find the answer to $\frac{4}{6}$ of 18 a group of students shared their ideas. For each student is their strategy correct? Explain why or why not.

- Max knew that 18 divided into 6 equal groups means that there are 3 in each group so the answer is 3 .
- Oprah drew 18 circles in rows of 6 . He shaded the first 4 circles in each row and counted 12 shaded circles
- Asher said, "I know that $6 \times 3=18$ so $\frac{1}{6}$ of 18 is 3 . I then need to find $\frac{4}{6}$ of 18 so I added 3 up four times to get 12 .
- Ezekiel skip counted by 6 until he reached 18. He realized that 18 was the 3rd multiple he said so the answer is 3 .


## Use unit fractions to understand operations of fractions.

NC.4.NF. 4 Apply and extend previous understandings of multiplication to:

- Model and explain how fractions can be represented by multiplying a whole number by a unit fraction, using this understanding to multiply a whole number by any fraction less than one.
- Solve word problems involving multiplication of a fraction by a whole number

| Clarification | Checking for Understanding |
| :--- | :--- |
|  | Possible response: |
|  | The correct answer is 12. Explanations should use arrays or other |
|  | models or reasoning about the relationship between the fraction and the |
|  | number of objects in the set. |

Return to Standards

## Understand decimal notation for fractions and compare decimal fractions.

NC.4.NF. 6 Use decimal notation to represent fractions.

- Express, model and explain the equivalence between fractions with denominators of 10 and 100
- Use equivalent fractions to add two fractions with denominators of 10 or 100.
- Represent tenths and hundredths with models, making connections between fractions and decimals.


## Clarification

This standard is the first time that students explore decimals. They are introduced to decimals through area models for fractions that are partitioned into 10 equal parts (tenths) and decimal grids which are $10 \times 10$ grids with 100 squares (hundredths). Students should have ample opportunities to explore and reason about the idea that a number can be represented as both a fraction and a decimal. This standard establishes the connection that a fraction that has been equally partitioned into 10 or 100 equal parts ( $10^{\text {th }}$ and 100ths) can also be written as a decimal.

With the second bullet of this standard students explore how to find an equivalent fraction for both tenths and hundredths to add two fractions. Students are expected to find the sum of two fractions using area models or physical manipulatives such as base-ten blocks. Money should not be used as a context since it is a non-proportional representation, meaning that a dime is not ten times larger than a penny.

In the third bullet, models should focus on area models partitioned in tenths or hundredths, and number lines. Students can also make connections between fractions with denominators of 10 and 100 and the place value chart.

By reading fraction names, students say $32 / 100$ as thirty-two hundredths and rewrite this as 0.32 or represent it on a place value model as shown below.

| Hundreds | Tens | Ones | $\bullet$ | Tenths | Hundredths |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  | $\bullet$ | 3 | 2 |

Checking for Understanding
Express, model, and explain the equivalence between fractions and represent tenths and hundredths with models
Rosita has $\frac{8}{10}$ of a meter of ribbon. However, the directions for her craft product have directions written about hundredths of a meter. What is an equivalent decimal to $\frac{8}{10}$ to the hundredths place?

Possible response:
I shaded in 8 columns on the decimal grid. That is the same as $\frac{80}{100}$ which can also be written as 0.8 or 0.80 .


Shade $\frac{4}{10}$ on a decimal grid. On a different decimal grid shade in $\frac{4}{100}$. Explain how the two fractions are different on the decimal grid. Explain how they are different when written as decimals using a place value chart.

Possible response:
I shaded both fractions on separate
decimal grids. The grid on the left shows
$\frac{4}{100}$ and the grid on the right shows $\frac{4}{10}$.
The fraction $\frac{4}{100}$ means 4 out of 100 and

since there are 100 small squares I only shaded in 4 of them. On the place value chart 4 out of 100 means that I have 4 hundredths so there is a 4 in the hundredths place and a 0 in the tenths place.


The fraction $\frac{4}{10}$ has 4 columns shaded since that fraction means 4 out of 10 and there are 10 columns. On the place value chart 4 out of 10 or $\frac{4}{10}$ is 4 tenths so there is a 4 in the tenths place.

Understand decimal notation for fractions and compare decimal fractions.
NC.4.NF. 6 Use decimal notation to represent fractions.

- Express, model and explain the equivalence between fractions with denominators of 10 and 100
- Use equivalent fractions to add two fractions with denominators of 10 or 100.
- Represent tenths and hundredths with models, making connections between fractions and decimals.

| Clarification | Checking for Understanding |
| :--- | :--- |

Use equivalent fractions to add two fractions with denominators of 10 and 100 Mitch swam $\frac{5}{10}$ of a mile on Saturday and $\frac{39}{100}$ a mile on Sunday. How much did Mitch swim on the two days? Use a decimal grid to show your answer and write your answer as a decimal.

Possible response:


89 of the one hundred squares are shaded so Mitch swam $\frac{89}{100}$ of a mile. I can also write $\frac{89}{100}$ as 0.89 .

## Understand decimal notation for fractions and compare decimal fractions.

NC.4.NF. 7 Compare two decimals to hundredths by reasoning about their size using area and length models, and recording the results of comparisons with the symbols $>,=$, or <. Recognize that comparisons are valid only when the two decimals refer to the same whole.

## Clarification

Students should reason that comparisons are only valid when they refer to the same whole. Comparisons should only be done in Grade 4 with area and length models, which include decimal grids, decima circles, number lines, and meter sticks. Students should be able to construct their own models

## Checking for Understanding

Sarah drinks 0.37 Liters of juice. Rochelle drinks 0.4 Liters of juice. Who drank more juice? Draw a picture and explain your reasoning.

Possible response:


Sarah


Rochelle

Sarah drank 37 hundredths of a Liter, while Rochelle drank 4 tenths or 40 hundredths of a Liter. Rochelle drank more.

Theresa has $\frac{4}{100}$ of a Liter of milk in one container and a brand new container. Her recipe has the number 0.4 of milk in it. Can she make the recipe with the milk in one container or does she need to open a new container?

Possible response:
She needs to open a new container since 0.4 is equal to $\frac{4}{10}$ and she has only $\frac{4}{100}$ of a Liter in her container.

Denard lives 0.67 km from school and Calvin lives 0.76 km from school. Draw a number line to show who lives the farthest from school.

Possible student response:
Denard lives 67 hundredths of a kilometer away which is less than 7 tenths of a kilometer. Calvin lives 76 hundredths of a kilometer away which is greater than 7 tenths. Calvin lives
 farther from school than Denard.

The decimal grid shows the amount of kilograms of turkey that Mrs. Burrell has to make sandwiches. Mrs. Rigamarole has more than 0.5 kilograms of turkey but less than Mrs. Burrell. How much turkey could Mrs. Rigamarole have?

Possible response:
Mrs. Rigamarole has more than 0.5 kilograms and less than 0.6 kilograms. She could have as few as 0.51 or as many as 0.59 .

## Measurement and Data

## Solve problems involving measurement.

NC.4.MD. 1 Know relative sizes of measurement units. Solve problems involving metric measurement.

- Measure to solve problems involving metric units: centimeter, meter, gram, kilogram, Liter, milliliter.
- Add, subtract, multiply, and divide to solve one-step word problems involving whole-number measurements of length, mass, and capacity that are given in metric units.


## Clarification

In this standard, students reason about the units of length, capacity and weight using metric units. Students need to develop a basic understanding of the size and weight of metric units and apply this understanding when estimating and measuring. Students should understand how to express larger measurements in smaller units within the metric system to reinforce notions of place value.

In this standard, word problems should only be one-step and include the same units. In Grade 4, students are not expected to do conversions between units before solving problems.

## Checking for Understanding

One can of soda holds 376 mL . A large container holds 8 times more punch than the can of soda. How much soda does the large container hold?

Possible response:
The large container holds $376 \times 8 \mathrm{~mL}$.

| 300 | 70 | 6 |
| :---: | :---: | :---: |
| $\mathbf{8} 300 \times 8=2,400$ | $70 \times 8=560$ | $6 \times 8=48$ |
|  |  | 2,400 <br> 560 <br> +48 <br> 3,008 |

The container holds 3,008 mL.

I have $4,327 \mathrm{~mL}$ of juice. I want to divide the juice equally into 8 containers. The remaining juice is all poured into one of the containers. How much juice will be in the container that has the most juice? How much juice will be in the other containers?

## Possible responses:

| sible respons |  | 500 |
| :---: | :---: | :---: |
| Student A | 40 | 500 +40 |
| 1 used partial quotients. The answer was 540 | 500 | $\frac{+40}{540}$ |
| with a remainder of 7 . That means that one | 84327 |  |
| containers had 540 mL of juice. | -4000 | ( $500 \times 8$ |
|  | 327 |  |
| Student B | -320 | $(40 \times 8)$ |
| I multiplied up using 8s. I knew | 7 |  |
| $5 \times 8=40$ |  |  |
| $50 \times 8=400$ |  |  |
| $500 \times 8=4,000$ |  |  |
| $4 \times 8=32$ |  |  |
| $40 \times 8=320$ so $540 \times 8=4,320$ with a rem | der of 7. |  |
| One container received 547 mL and the rest | hem had 54 |  |

## Solve problems involving measurement.

NC.4.MD. 2 Use multiplicative reasoning to convert metric measurements from a larger unit to a smaller unit using place value understanding, two-column tables, and length models.

## In this standard, students should understand how to express larger

 measurements in smaller units within the metric system to reinforce notions of place value. Students will make metric conversions from larger units to smaller units exploring the relationship between the units. Students may use a twocolumn chart to convert from larger to smaller units and record equivalent measurements.Through exploration with place value and conversions, students may explore with various metric prefixes. However, students are only responsible for knowing conversions between centimeter/meter, gram/kilogram, and Liter/milliliter.

Students are also expected to use their place value understanding to support conversions. For example, since 1 meter is 100 centimeters, the number of centimeters is 100 times the number of meters. Therefore, if a piece of rope is 3 meters, then the rope is 300 cm long.

This standard is focused on whole number conversions of metric units since students do not multiply decimals until Grade 5

## Checking for Understanding

Crystal has 8 Liters of soda. How many milliliters does she have?
Possible response:
There are $1,000 \mathrm{~mL}$ in 1 Liter. So, in 8 Liters the number of $m L$ is $8 x$ 1,000 which is 8,000 .

Complete the table below:

| Meters | Centimeters |
| ---: | :--- |
| 3 |  |
| 4 |  |
| 5 |  |
| 12 |  |
| 120 |  |

Answers: $3 \mathrm{~m}=300 \mathrm{~cm}, 4 \mathrm{~m}=400 \mathrm{~cm}, 5 \mathrm{~m}=500 \mathrm{~cm}, 12 \mathrm{~m}=1,200 \mathrm{~cm}$, $120 \mathrm{~m}=12,000 \mathrm{~cm}$

Charlotte tells her teacher that a container that holds 2 Liters holds 200 mL of liquid. Is she correct or incorrect? Explain why.

Possible Response:
Since 1 Liter equals 1000 mL, 2 Liters is equal to $2,000 \mathrm{~mL}$. Charlotte is incorrect since she said 2 Liters equals 200 mL

## Solve problems involving measurement.

NC.4.MD. 8 Solve word problems involving addition and subtraction of time intervals that cross the hour.

## Clarification

## Checking for Understanding

In this standard, students apply addition and subtraction strategies to find an end time, amount of time passed, or a start time. In third grade, students determined elapsed time within an hour. This standard calls for students to be able to cross over the hour. Students should use tools such as clocks, timelines, and tables to solve problems.

This standard could include situations with two activities that include time that passed (or duration), which may result in a multistep problem.

## One-step word problem

The movie started at 2:30 pm and lasted for 1 hour and 35 minutes. What time did the movie end?

## Possible response:

Student A:
I started at 2:30 and added 1 hour and moved to $3: 30$. I then added 30 minutes and moved to 4:00 p.m. I
then had to move 5 more minutes which was 4:05 p.m.

Student B:


I decomposed 1 hour 35 minutes into 1 hour +30 minutes +5 minutes.
$2: 30+1$ hour +35 minutes.
$2: 30+30$ minutes $=3: 00$
3:00 + 5 minutes $=3: 05$
3:05 + 1 hour $=4: 05$ p.m.

## Multi-step word problem

I wake up at 6:47 a.m. I get ready for school which takes 15 minutes and then eat breakfast before leaving. If I leave for school at 7:21 a.m. how long do I have to eat breakfast?


Possible response:
I added 15 minutes to 6:47 by moving 13 minutes until 7:00 and then moving 2 minutes until 7:02. I then knew I had to eat breakfast until 7:21. In order to find this amount, I added up 8 until 7:10 and then added 11 more to get to 7:21 My answer is the distance between 7:02 to 7:21, which is $8+11$ or 19 minutes.

## Solve problems involving area and perimeter.

NC.4.MD. 3 Solve problems with area and perimeter.

- Find areas of rectilinear figures with known side lengths.
- Solve problems involving a fixed area and varying perimeters and a fixed perimeter and varying areas.
- Apply the area and perimeter formulas for rectangles in real world and mathematical problems.


## Clarification

In this standard, students will apply their previous understanding of perimeter and area to problem situations.

Students will be able to determine the area of a rectilinear figure. A rectilinear figure is a polygon that has all right angles. Recognizing that area is additive, students will be able to decompose the rectilinear figure into rectangles, determine the area of the rectangles, and use the areas of the rectangles to determine the area of the rectilinear figure.

Students will solve problems that involve exploration of the relationship between perimeter


How could this figure be decomposed to help find the area?


This portion of the decomposed fig a $4 \times 2$.

This portion of the decomposed figure is $2 \times 2$.
$4 \times 2=8$ and $2 \times 2=4$
$8+4=12$
The total area of this figure is 12 square units and area in a rectangle. When given a fixed area, students will be able to determine all of the possible dimensions of the rectangle. When given a fixed perimeter, students will be able to determine all possible areas.

Students learn to apply these understandings and formulas to the solution of real-world and mathematical problems. Note that "apply the formula" does not mean write down a memorized formula and put in known values. In fourth grade, working with perimeter and area of rectangles is still based in models and strategies.

## Checking for Understanding

## Find areas of rectilinear figures with known side lengths

Mr. Rutherford is covering the miniature golf course with an artificial grass. How many 1 -foot squares of carpet will he need to cover the entire course?


Possible Response:

## Student A

decided to break the shape into 3 rectangles. The one on the left was 5 feet long and 2 feet wide which is $5 \times 2$ or 10 square feet. The middle rectangle was 3 feet long and 2 feet wide which is $3 x 2$ or 6 square feet. The right rectangle was 4 feet long and 2 feet wide which is $4 \times 2$ or 8 square feet. I add them together to get a total of $10+6+8$ or 24 square feet.

Student B
I counted 24 squares, so the area is 24 square feet.

A storage shed is pictured to the right. What is the total area?
How could the figure be decomposed to help find the area?

Possible Response:
I decomposed the shape into three rectangles to find the area of each rectangle. The top rectangle is 10 wide $x$


5 long or 50 square meters. The middle
rectangle has a width of $10-6=4$ and a length of 5 so the area is $5 \times 4$ or 20 square meters. The bottom rectangle is $10 \times 5$ or 50 square meters.
The total area is $50+20+50$ which is 120 square meters.

## Solve problems involving area and perimeter.

NC.4.MD. 3 Solve problems with area and perimeter.

- Find areas of rectilinear figures with known side lengths.
- Solve problems involving a fixed area and varying perimeters and a fixed perimeter and varying areas.
- Apply the area and perimeter formulas for rectangles in real world and mathematical problems


## Clarification

## Checking for Understanding

Solve problems involving a fixed perimeter and varying areas
At Miguel's apartment complex they are building a rectangular outdoor eating space with a perimeter of 32 meters. Side lengths will be whole numbers and each dimension is less than 13 meters. What are the possible areas? Which dimensions have the most space for people to eat?

Possible Response:
I made a table of rectangle dimensions with the perimeter of 32 and different areas. The dimensions of $8 \times 8$, which is a square, gives people the most space to eat.

| Dimensions | Perimeter | Area |
| :---: | :---: | :---: |
| $8 \times 8$ | 32 | 64 |
| $9 \times 7$ | 32 | 63 |
| $10 \times 6$ | 32 | 60 |
| $11 \times 5$ | 32 | 55 |
| $12 \times 4$ | 32 | 48 |

Solve problems with a fixed area and varying perimeters
You want to build a region that has an area of 12 square meters. What are the possible dimensions? Which dimensions require the least amount of fencing?

Possible solution:

| Area | Length | Width | Perimeter |
| :---: | :---: | :---: | :---: |
| 12 sq. m | 1 m | 12 m | 26 m |
| $12 \mathrm{sq} . \mathrm{m}$. | 2 m | 6 m | 16 m |
| 12 sq. m | 3 m | 4 m | 14 m |
| 12 sq. m | 4 m | 3 m | 14 m |
| 12 sq. m | 6 m | 2 m. | 16 m |
| 12 sq. m | 12 m | 1 m | 26 m |

## Represent and interpret data.

NC.4.MD. 4 Represent and interpret data using whole numbers.

- Collect data by asking a question that yields numerical data.
- Make a representation of data and interpret data in a frequency table, scaled bar graph, and/or line plot.
- Determine whether a survey question will yield categorical or numerical data


## Clarification

In this standard, students will interact with data by posing a question, interacting with data, analyzing data, and interpreting data.

Students need ample experiences with creating and discussing survey questions, data collection creating scaled bar graphs and line plots, and interpretating data. In third grade, students collected data by asking a question that yielded categorical data, which is data that can be grouped into categories. Students in fourth grade will build on that concept and begin to also ask questions that provide numerical data, which is data that is measurable such as time, height, weight, temperature, etc.

Once data is collected, students should be able to choose an appropriate representation of categorical or numerical data and create the representation. Students will create frequency tables, scaled bar graphs or line plots based on the data collected. Graphs should include a title, categories, category label, key, and data. Once graphs are created, students should be able to solve simple one and two-step problems using the information in the graphs.

In Grade 3, students learned that scaled bar graphs have a scale on the $y$-axis in which the labels do not include every number.

## Checking for Understanding

## Make a representation of data and interpret data in a line plo

Mrs. Smith's class tracked the daily high temperatures for 20 days in July. The chart below shows the data that the class collected.

| 90 | 92 | 93 | 92 | 89 |
| :--- | :--- | :--- | :--- | :--- |
| 93 | 91 | 95 | 88 | 90 |
| 95 | 94 | 97 | 97 | 94 |
| 94 | 91 | 90 | 89 | 94 |

a. Create a line plot that shows the frequency of the July high temperatures. Make sure you label the scale.
b. If the normal daily high temperature for July is $90^{\circ}$, how many days was the high temperature less than normal?
c. What was the most frequent daily high temperature recorded during the 20 days?

Possible Response:


There were 3 days where the temperature was below 90 degrees or less than normal.
The most frequent daily high temperature was 94 degrees.

Students were given the choice of eating a hot dog, a cheeseburger, or a chicken sandwich at the class picnic. The data is in the table to the right.

There were some students who did not vote. The number of students who voted for cheeseburger is 3 times the number of students who did not vote.

Make a scaled bar graph that shows the data. Include the nonvoters in the graph

| Food | Number of Students |
| :--- | :--- |
| Hot Dog | 9 |
| Cheeseburger | 6 |
| Chicken Sandwich | 5 |

## Represent and interpret data.

NC.4.MD. 4 Represent and interpret data using whole numbers.

- Collect data by asking a question that yields numerical data.
- Make a representation of data and interpret data in a frequency table, scaled bar graph, and/or line plot.
- Determine whether a survey question will yield categorical or numerical data.


## Clarification

## Checking for Understanding

## Possible response:

There were 2 non-voters since 6 is 3 times more than 2 .

## Class Picnic



Determine whether a survey question will yield categorical or numerical data
For each question determine whether the responses will be categorical data or numerical data:
How many hours did people sleep last night?
Would you rather hike, swim, or play soccer today?
How many times can you clap your hands in 10 seconds?
At a picnic would you want to eat a hot dog, a cheeseburger, or a salad?
Answers:
Hours of sleep- Numerical
Outdoor activity- Categorical
Clapping hands- Numerical
Picnic- Categorical

## Understand concepts of angles and measure angles

NC.4.MD. 6 Develop an understanding of angles and angle measurement.

- Understand angles as geometric shapes that are formed wherever two rays share a common endpoint and are measured in degrees.
- Measure and sketch angles in whole-number degrees using a protractor.
- Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems.


## Clarification

In this standard, students explore angles and their properties. An angle is formed by two rays that share an endpoint and is measured with reference to the degrees of a circle.

For example: If the common endpoint of two rays is the center of a circle, the angle can be measured by considering the fraction of the circular arc between the points where the rays intersect the circle.


An angle that turns $\frac{1}{360}$ of a circle is a "one-degree angle".

Students will be able to identify three types of angles: right angle, acute angle, and obtuse angle

|  | right angle: <br> An angle that equals one quarter of a full <br> rotation of a circle, or $90^{\circ}$ |
| :--- | :--- |
| acute angle: |  |
| An angle that is less than a right angle, or |  |
| less than $90^{\circ}$ |  |$|$| obtuse angle: |
| :--- |
| An angle that is more than a right angle, or |
| more than $90^{\circ}$. |

## Checking for Understanding

Measure and sketch angles in whole-number degrees using a protractor Draw Angle ABC that is 30 degrees.
Explain whether Angle $A B C$ is an acute, an obtuse, or a right angle.
Now draw Angle DEF that is 3 times larger than Angle ABC.
Explain whether Angle DEF is an acute, an obtuse, or a right angle.
Now draw Angle GHI that is 5 times larger than Angle ABC.
Explain whether Angle GHI is an acute, an obtuse, or a right angle.
Answers:
$A B C$ is an acute angle that is 30 degrees
$D E F$ is a right angle that is 90 degrees
GHI is an obtuse angle that is 150 degrees.

Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems
If the two rays are perpendicular, what is the value of $m$ ?


Possible Response:
The large angle has a measure of 90 degrees so Angle $M+25+20=$ 90. I can subtract both 25 and 20 from 90 which means Angle M is 45 degrees.

A lawn water sprinkler rotates 85 degress and then pauses. It then rotates an additional 19 degrees. If it does this 3 times, what is the total degree of the water sprinkler rotation?

Possible Response:
The water sprinkler goes 68 degrees and then another 19 degrees. That is a total of 87 degrees. Since it does this 3 times that is a total of $87 \times 3=$ 261 degrees.

North Carolina Department of

## Understand concepts of angles and measure angles.

NC.4.MD. 6 Develop an understanding of angles and angle measurement.

- Understand angles as geometric shapes that are formed wherever two rays share a common endpoint and are measured in degrees.
- Measure and sketch angles in whole-number degrees using a protractor.
- Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems.


## Clarification

Students will learn to measure and sketch angles using a protractor. Students should also have an understanding of benchmark angles such as $45^{\circ}, 90^{\circ}$ and $180^{\circ}$.

## For example:

When measuring angles with a protractor, students will first identify if the angle is acute or obtuse to determine which numbers to use. Acute angles would measure $0^{\circ}$ to $89^{\circ}$, and obtuse angles would measure $91^{\circ}$ to $179^{\circ}$. In this example, the angle is obtuse, so it would be read as $120^{\circ}$.


In the following example, the angle measured is obtuse, but facing the opposite direction of the angle pictured to the left. Students who understand that obtuse angles measure $91^{\circ}$ to $179^{\circ}$ would know that this angle measures $135^{\circ}$ rather than $45^{\circ}$.


Students will explore the additive nature of angle measurements and solve one- and two-step addition and subtraction problems where an angle measure is missing. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts.

In terms of vocabulary, students explore the concepts of complementary and supplementary angles, but are not responsible for these terms. Two angles are called complementary if their measurements have the sum of $90^{\circ}$. Two angles are called supplementary if their measurements have the sum of $180^{\circ}$. Two angles with the same vertex that share a side are called adjacent angles. These terms may come up in classroom discussion, but students are not responsible for knowing these terms. These concepts are developed thoroughly in middle school ( $7^{\text {th }}$ grade).

## Checking for Understanding

Joey knows that when a clock's hands are exactly on 12 and 1, the angle formed by the clock's hands measures $30^{\circ}$. What is the measure of the angle formed when a clock's hands are exactly on the 12 and 4 ?

## Possible Response:

Student A
12 to 1 is 30 degrees. 1 to 2 is another 30 degrees. 2 to 3 is another 30 degrees. 3 to 4 is another 30 degrees.
$30+30+30+30=120$ degrees

## Student B

12 to 1 is a rotation of 30 degrees. 12 to 4 is 4 times larger. $30 \times 4$ is 120 so the angle is 120 degrees when the hands are at 12 and 4.

## Classify shapes based on lines and angles in two-dimensional figures.

## NC.4.G. 1 Draw and identify points, lines, line segments, rays, angles, and perpendicular and parallel lines.

## Clarification

This standard asks students to draw two-dimensional geometric objects and to also identify them in two-dimensional figures.

Students should be able to draw and identify the following figures:
point

Students should understand the concept of parallel and perpendicular lines. Two lines are parallel if they never intersect and are always equidistant. Two lines are perpendicular if they intersect in right angles $\left(90^{\circ}\right)$.

Lines $A B$ and $C D$ are parallel. Line $F G$ is perpendicular to lines $A B$ and $C D$ forming right angles.


## Checking for Understanding

Draw the following shapes:
Angle $A B C$ is a straight angle.
Ray BD intersects Angle ABC and Angle ABD into a right angle.
Ray BE intersects Angle ABC and Angle EBC is a 30 degree angle.
What is the measure of Angle DBE?
Possible Response:


Draw two different types of quadrilaterals that have two pairs of parallel sides. Describe what the two shapes have in common. Describe differences between the two shapes.

Possible responses:
Shapes may include a square, rectangle, rhombus, parallelograms that are not squares. Descriptions discuss the similarity of 4 sides, 2 pairs of parallel sides. Differences may include the types of angles and the length of the sides.

Is it possible to have a right triangle with an obtuse angle? Justify your reasoning using pictures and words.

Possible response:
A right triangle has a 90-degree angle and the 2 other angles must be acute. Therefore, any triangle with a right angle can never have an obtuse angle.

Draw and list the properties of a parallelogram. Draw and list the properties of a rectangle. How are your drawings and lists alike? How are they different? Be ready to share your thinking with the class.

## Classify shapes based on lines and angles in two-dimensional figures.

NC.4.G. 1 Draw and identify points, lines, line segments, rays, angles, and perpendicular and parallel lines.

| Clarification | Checking for Understanding |
| :--- | :--- |
|  | Possible response: <br> A parallelogram is a quadrilateral with 2 pairs of parallel sides. The <br> opposite sides are the same length. A rectangle has all of those |
|  |  |
|  | and | opposite sides are the same length. A rectangle has all of those characteristics. In addition, a rectangle always has 4 right angles.

## How many acute, obtuse and right angles are in this shape? Explain how you

 know. Now draw a shape that has the same name that has 2 right angles.

Possible response:
This shape has 2 acute and 2 obtuse angles. The trapezoid that has 2 right angles is called a right trapezoid. I drew it here.


Return to Standards

## Classify shapes based on lines and angles in two-dimensional figures.

NC.4.G.2 Classify quadrilaterals and triangles based on angle measure, side lengths, and the presence or absence of parallel or perpendicular lines.

## Clarification

This standard calls for students to sort and classify quadrilaterals and triangles based on parallelism, perpendicularity, the lengths of sides, and angle types.

Students should be able to use side length to classify triangles as equilateral, isosceles, or scalene; and can use angle size to classify them as acute, right, or obtuse. They then learn to cross-classify, for example, naming a shape as a right isosceles triangle.

Students should be able to use the relationships between sides (parallel and perpendicular) and side lengths to classify quadrilaterals. Students are expected to know the characteristics of: quadrilateral, trapezoid, parallelogram, rectangle, square, and rhombus. While working with quadrilaterals, students' experiences with drawing and identifying right, acute, and obtuse angles support them in classifying two-dimensional figures based on specific angle measurements. They use the benchmark angles of $90^{\circ}$, $180^{\circ}$, and $360^{\circ}$ to approximate the measurement of angles. Students are not expected to know the sum of the interior angles of a triangle or a quadrilateral in Grade 4.

For example: The square has perpendicular lines because the sides meet at a corner, forming right angles. It also has parallel sides that are opposite from each other. I know this because if I changed the sides to lines that never end, the lines would never intersect and be the same distance apart. Segments are just parts of lines. All of the line segments are equal.


The notion of congruence ("same size and same shape") may be part of classroom conversation, but the concepts of congruence and similarity do not appear in standards until middle school.

Note: North Carolina has adopted the exclusive definition for a trapezoid. A trapezoid is a quadrilateral with exactly one pair of parallel sides.

Checking for Understanding
Do you agree with the label on each of the circles in the Venn diagram above? Describe why some shapes fall in the overlapping sections of the circles.


Add one more figure to the diagram. Explain where it should go and why.
Possible response:
I agree with the labels on the Venn Diagram.
The shapes in the center section have at least one set of parallel sides and at least one right angle.

Adding two more figures:
A trapezoid with 2 right angles could be added to the center section since it has 2 right angles and has at least one set of parallel sides.

For each of the following, sketch an example if it is possible. If it is impossible, say so, and explain why or show a counter example.

- A parallelogram with exactly one right angle. (impossible)
- An isosceles right triangle.
- A rectangle that is not a parallelogram. (impossible)
- A square that is a rhombus
- A trapezoid that is a parallelogram.
- A parallelogram that has 4 right angles.

Possible responses:

## Isosceles right triangle:



Every square is a rhombus since it has 4 congruent sides.
A parallelogram that has 4 right angles can be a rectangle or a square.

## Classify shapes based on lines and angles in two-dimensional figures.

NC.4.G. 2 Classify quadrilaterals and triangles based on angle measure, side lengths, and the presence or absence of parallel or perpendicular lines.
Checking for Understanding
Identify which of these shapes have perpendicular or parallel sides and justify your selection


Square: 2 pairs of parallel sides. 4 pairs of perpendicular sides that form 4 right angles.
Triangle: No parallel and no perpendicular sides.
Pentagon: No parallel and no perpendicular sides.
Trapezoid: Exactly 1 pair of parallel sides and no pairs of perpendicular sides.

Return to Standards

## Classify shapes based on lines and angles in two-dimensional figures.

NC.4.G. 3 Recognize symmetry in a two-dimensional figure, and identify and draw lines of symmetry.

## Clarification

## Checking for Understanding

Do these figures have lines of symmetry? If so, identify the lines of symmetry.

## TH S D

 symmetrical how many lines of symmetry are in a figure. Students explore the concept that a line of symmetry is a line that divides a figure into two parts that are identical in shape and size.Students are expected to identify and draw lines of symmetry in regular and non-regular polygons, circles, and letters of the alphabet. Circles have an infinite number of lines of symmetry while all regular and non-regular polygons have a specific number of lines of symmetry. Students are expected to understand that some figures may have more than one line of symmetry. In Grade 4, students only explore line symmetry not rotational symmetry.


The triangle has one line of symmetry, and the square has four lines of symmetry.


Possible response:
This 1 vertical line of symmetry through the center.
$H$ has 1 vertical line of symmetry through the center.
S has 0 lines of symmetry.
D has 1 horizontal line of symmetry through the center.
Explain how the number of lines of symmetry differ in a square and a rectangle that is not a square.

Possible response:
A square has 4 lines of symmetry- a vertical line through the center, a horizontal line through the center, and 2 diagonal lines from upper left to bottom right and upper right to bottom left through the center. A rectangle that is not a square has 2 lines of symmetry- a vertical line through the center, a horizontal line through the center.

Explain what has more lines of symmetry: a circle or a square.
Possible response:
A circle has more lines of symmetry than a square. A square has 4 lines of symmetry. A circle has an infinite number of lines of symmetry through the center of the circle.

