North Carolina Department of Public Instruction
INSTRUCTIONAL SUPPORT TOOLS
FOR ACHIEVING NEW STANDARDS

Discrete Mathematics for Computer Science ● Unpacked Contents
For the new Standard Course of Study that will be effective in all North Carolina schools in the 2020-21 School Year.

This document is designed to help North Carolina educators teach **Discrete Mathematics for Computer Science** Standard Course of Study. NCDPI staff are continually updating and improving these tools to better serve teachers and districts.

What is the purpose of this document?
The purpose of this document is to increase student achievement by ensuring educators understand the expectations of the new standards. This document may also be used to facilitate discussion among teachers and curriculum staff and to encourage coherence in the sequence, pacing, and units of study for grade-level curricula. This document, along with on-going professional development, is one of many resources used to understand and teach the NC SCOS.

What is in the document?
This document includes a detailed clarification of each standard in the grade level along with a *sample* of questions or directions that may be used during the instructional sequence to determine whether students are meeting the learning objective outlined by the standard. These items are included to support classroom instruction and are not intended to reflect summative assessment items. The examples included may not fully address the scope of the standard. The document also includes a table of contents of the standards organized by domain with hyperlinks to assist in navigating the electronic version of this instructional support tool.

How do I send Feedback?
Link for: Feedback for NC's Unpacking Documents.
We will use your input to refine our unpacking of the standards. Thank You!

Just want the standards alone?
Link to: North Carolina Mathematics Standards

Discrete Mathematics for Computer Science Standards

[Standards for Mathematical Practice](#)

Number & Quantity	Functions	Statistics & Probability	Graph Theory	Logic
DCS.N.1 *Apply operations with matrices and vectors.* DCS.N.1.1 DCS.N.1.2 DCS.N.1.3 ***DCS.N.2*** *Understand matrices to solve problems.* DCS.N.2.1 DCS.N.2.2 DCS.N.2.3 DCS.N.2.4 ***DCS.N.3*** *Understand set theory to solve problems.* DCS.N.3.1 DCS.N.3.2 DCS.N.3.3 DCS.N.3.4 ***DCS.N. 4*** *Understand statements related to number theory and set theory.* DCS.N.4.1 DCS.N.4.2 DCS.N.4.3 DCS.N.4.4	***DCS.F.1*** *Apply recursively-defined relationships to solve problems.* DCS.F.1.1 DCS.F.1.2 DCS.F.1.3 DCS.F.1.4 DCS.F.1.5	***DCS.SP.1*** *Apply combinatorics concepts to solve problems.* DCS.SP.1.1 DCS.SP.1.2	***DCS.GT.1*** *Understand graph theory to model relationships and solve problems.* DCS.GT.1.1 DCS.GT.1.2 DCS.GT.1.3 ***DCS.GT.2*** *Apply graph theory to solve problems.* DCS.GT.2.1 DCS.GT.2.2 DCS.GT.2.3 DCS.GT.2.4	***DCS.L.1*** *Evaluate mathematical logic to model and solve problems.* DCS.L.1.1 DCS.L.1.2 DCS.L.1.3 DCS.L.1.4

Standards for Mathematical Practice

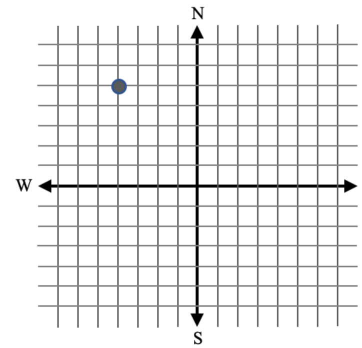
Practice	Explanation and Example
1. Make sense of problems and persevere in solving them.	In Discrete Mathematics for Computer Science (DCS), students solve real world problems through the application of theory and algorithmic thinking. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?”, and “Can I solve the problem in a different way?”
2. Reason abstractly and quantitatively.	In DCS, students represent a wide variety of real world contexts through the use of matrices, sets, diagrams, vertex-edge graphs and tables. They examine patterns in their processes. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
3. Construct viable arguments and critique the reasoning of others.	In DCS, students construct arguments using verbal or written explanations accompanied by matrices, expressions, equations, graphs, and tables. They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like “How did you get that?”, “Why is that true?” “Does that always work?” They explain their thinking to others and respond to others’ thinking.
4. Model with mathematics.	In DCS, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students solve systems of equations using matrices. Students use vertex-edge graphs to represent data and describe associations between variables. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.
5. Use appropriate tools strategically.	Students consider available tools when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in DCS may create a minimum spanning tree to create an optimal schedule. Like hand tools, it is essential for students to know when and how to use the many algorithms that are a part of this course. It is essential for students to make a choice between methods that produce optimal solutions and those that can be solved efficiently.
6. Attend to precision.	In DCS, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to the matrices, vectors, set and number theory, graph theory, and logical reasoning.
7. Look for and make use of structure.	Students routinely seek patterns or structures to model and solve problems. In DCS, students apply properties to generate equivalent expressions that involve matrices, sets, recursion, and logic statements.
8. Look for and express regularity in repeated reasoning.	In DCS, students use repeated reasoning to understand algorithms and make generalizations about patterns. Students use iterative processes to determine the GCF and LCM between any two numbers and use a variety of algorithms to solve problems in a context.
9. Use strategies and procedures flexibly.	Students make a choice between methods and algorithms that produce optimal solutions and those that can be solved efficiently in DCS. It is essential for a student to not only know an algorithm, but to know when best to use that particular algorithm. Students should be comfortable solving a problem, using the same or different algorithms, and producing different solutions that are valid.
10. Reflect on mistakes and misconceptions.	In DCS, it is essential for students to reflect upon mistakes and misconceptions. Mistakes are often the cornerstone of learning. Successful students in this course will reflect upon their own thinking and learning to maximize their potential to find optimal solutions efficiently.

Return to: Standards

DCS.N.1 Apply operations with matrices and vectors. DCS.N.1.1 Implement procedures of addition, subtraction, multiplication, and scalar multiplication on matrices.	
Clarification	**Checking for Understanding**
Students have not been introduced to matrices in previous courses. However, the concept of a matrix is similar to two way tables seen in 8th grade. In the objective, students are expected to add, subtract, and multiply matrices in a context. This context can include, but is not limited to, real-world problems, the transformation of points on a coordinate plane, Leslie Models and Markov Chains. Technology should be used as deemed appropriate by the teacher. This objective works in conjunction with DCS.N.2.1 as students must decide how to organize data from the context before performing matrix operations. For students to make this decision, students must: ● understand matrix operations, ● be able to identify elements within a matrix and corresponding elements between matrices, ● name a matrix using rows and columns to determine the feasibility of operations, and ● know which properties of operation apply to matrices. For both Leslie Models and Markov Chains, students should be able to organize the data into the appropriate matrices (DCS.N.2.1). In alignment with this standard, students evaluate matrices to solve problems involving Leslie Models and Markov chains. For all evaluated matrix expressions, students will also interpret solutions in context (DCS.N.2.2).	**Formative Check:** Without the aid of technology, what is element e_{23} of the product of $G \cdot H$? $G = \begin{bmatrix} 3 & 2 \\ 5 & 0.5 \end{bmatrix}$ $H = \begin{bmatrix} 7 & -1 & 0.5 \\ -5 & -3.5 & 4 \end{bmatrix}$ *Answer: 4.5* **Indicator:** A few local companies donate spirit items which can be sold along with the items made by the Booster Club at games. JJ's Sporting Goods donates 100 hats and 100 pennants in September and 125 hats and 75 pennants in October. Friendly Fred's Foods donates 105 hats and 125 pennants in September and 110 hats and 100 pennants in October. Use matrices to show many items are available each month from both sources? *Answer: Student matrices may be organized differently.* $\begin{array}{c c} & \begin{array}{cc} \text{Hats} & \text{Pennants} \end{array} \\ \begin{array}{c} \text{Sept} \\ \text{Oct} \end{array} & \begin{bmatrix} 205 & 225 \\ 235 & 175 \end{bmatrix} \end{array}$ **Indicator**: Riverside High School Booster Club is running the concession stands at a JV Football game. ● Booster club sells cans of soda for \$1, a slice of pizza for \$3, a chicken sandwich for \$5, and candy for \$1.50. ● The home side concession stand sold 160 cans of soda, 82 slices of pizza, 103 chicken sandwiches, and 72 pieces of candy. ● The visitors side concession stand sold 102 cans of soda, 65 slices of pizza, 52 chicken sandwiches, and 37 pieces of candy. ● The booster club purchased the soda for \$.25 per can, each slice of pizza for \$1.5, each chicken sandwich for \$3.29, and each piece of candy for \$.70. ● The home side concession was stocked with 200 cans of soda, 96 slices of pizza, 120 chicken sandwiches, and 150 pieces of candy. ● The visitors side concession stand was stocked with 120 cans of soda, 80 slices of pizza, 80 chicken sandwiches, and 100 pieces of candy. Using matrices, determine the booster club's profit for the JV game. *Answer: \$464.49*

Return to: Standards

DCS.N.1 Apply operations with matrices and vectors.
DCS.N.1.2 Implement procedures of addition, subtraction, and scalar multiplication on vectors.

Clarification	**Checking for Understanding**
This is the student's introduction to vectors. In a science context, a vector often represents a force being applied in a certain direction. As this objective requires students to work with vectors in a context, it is important for students to properly interpret the components of a vector and distinguish a vector from coordinate points. The use of vectors can be connected to the students previous work with translations from 8th grade and NC Math 2. Only the component form of a vector will be used in this course. Students are expected to translate the vector into a matrix and use matrix operations to solve problems. A vector can be defined differently, depending on the context. For this standard, the definition of a vector is aligned to the mathematical and physical science definition. In computer programming, graphics, and security, the term vector has distinct but related definitions.	**Indicator**: A boat is traveling in water with a current that is flowing on average 2 mph directly west. The boat is traveling slowly on the vector $< 5, \ -3 >$ in mph. The graph, with a scale of 1 mile, indicates the boat's current position. A. Using matrices, write an expression that represents the position of the boat in *n* hours. B. Where will the boat be in 2 hours?  *Answer: a. Sample answer:* $\begin{bmatrix}-4\\5\end{bmatrix} + n\begin{bmatrix}5\\-3\end{bmatrix} - n\begin{bmatrix}2\\0\end{bmatrix}$ *b. (2,-1)* **Indicator**: The image of a triangle is A'(-2,1), B'(0,-1), and C'(-3,2). Triangle ABC was translated by $j <-2, 3>$. What are the coordinates of the vertices of the pre-image? *Answer: A(0,-2), B(2,-4), C(-1,-1)*

Return to: Standards

DCS.N.1 Apply operations with matrices and vectors.
DCS.N.1.3 Implement procedures to find the inverse of a matrix.

Clarification	**Checking for Understanding**
In the past, students have worked with additive and multiplicative inverses when solving equations. Students should be able to use the determinant to identify if the inverse of a matrix is possible. Students should be familiar with using technology to find the inverse of larger matrices.	**Indicator**: You receive the name of a famous piece of art hidden in the coded message: 21, 0, 53, 2, 11, 3, 24, 1. You know the original coding matrix, shown here. $\begin{bmatrix}-3 & 5\\-1 & 2\end{bmatrix}$ What matrix would be used to decode the message? *Answer:* $\begin{bmatrix}-2 & 5\\-1 & 3\end{bmatrix}$

The expectation of this standard is that students will discern the need to find the inverse matrix from a context.
Technology should be used as deemed appropriate by the teacher.

Indicator: During a soccer season, referees are paid different rates for different types of games. There are three types of games in a typical season: non-conference, conference, and playoff games. There are two referees for each game and schools only have to pay for home games. The information below is from three high schools with the same pay scale for referees.

High School	Home Games Nonconference	Home Games Conference	Playoff Games	Total Pay for Soccer Referees
Green River	5	7	2	$1806
Blue Creek	3	8	1	$1570
Black Lake	6	6	0	$1476

a. Write a matrix equation based on the context.
b. What is the inverse matrix you would use to solve the matrix equation.

Answers: a. $\begin{bmatrix} 5 & 7 & 2 \\ 3 & 8 & 1 \\ 6 & 6 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1806 \\ 1570 \\ 1476 \end{bmatrix}$, *b.* $\begin{bmatrix} \frac{1}{8} & -\frac{1}{4} & \frac{3}{16} \\ -\frac{1}{8} & \frac{1}{4} & -\frac{1}{48} \\ \frac{5}{8} & -\frac{1}{4} & -\frac{19}{48} \end{bmatrix}$

Return to: Standards

DCS.N.2 Understand matrices to solve problems.
DCS.N.2.1 Organize data into matrices to solve problems.

Clarification

In 8th grade, students organize data and solve problems using two-way tables. While the terminology of matrices will be new to students, the organizational structure is not. As with two-way tables, there are a variety of ways to organize data into a matrix, therefore, it is essential that students sufficiently label their data.

An essential part of the decision of how to organize data into matrices is the understanding of the requirements for matrix operations. In problems with multiple matrices that require matrix operations, students must understand that the choice of how to organize one matrix often then determines the organization of the other matrices in order to complete the operations.

Checking for Understanding

Indicator: Riverside High School Booster Club is running the concession stands at a JV Football game.

- Booster club sells cans of soda for $1, a slice of pizza for $3, a chicken sandwich for $5, and candy for $1.50.
- The home side concession stand sold 160 cans of soda, 82 slices of pizza, 103 chicken sandwiches, and 72 pieces of candy.
- The visitors side concession stand sold 102 cans of soda, 65 slices of pizza, 52 chicken sandwiches, and 37 pieces of candy.
- The booster club purchased the soda for $.25 per can, each slice of pizza for $1.5, each chicken sandwich for $3.29, and each piece of candy for $.70.
- The home side concession was stocked with 200 cans of soda, 96 slices of pizza, 120 chicken sandwiches, and 150 pieces of candy.
- The visitors side concession stand was stocked with 120 cans of soda, 80 slices of pizza, 80 chicken sandwiches, and 100 pieces of candy.

Students should also know how to organize data into matrices for Leslie Models and Markov Chains.

<u>Leslie Models</u>

Leslie Models are used to model population growth using age distributions with corresponding birth and survival rates.

The Leslie model is $P_k = P_0 \cdot L^k$, where P_0 is the initial population matrix, L is the Leslie matrix, and k is the number of cycles.

For this standard, students are expected to organize data into the initial population matrix and Leslie matrix.

<u>Markov Chains</u>

Markov Chains are used in situations that involve a finite number of states that change over time.

A Markov chain is $D_k = D_0 \cdot T^k$, where D_0 is the initial-state matrix, T is the transition matrix, and k is the number of transitions.

For this standard, students are expected to organize data into the initial-state matrix and transition matrix.

For this course, there are no set limits for the size of matrices or the number of matrices created to solve a problem.

Setup a matrix expression that represents the booster club's profits for the JV football game.

Answer: Student matrices may be organized differently. In general, when written as one expression: $P(H_s + V_s) - C(H_I + V_I)$*, where P represents the price of the item,* H_s *represents the items sold on the home side,* V_s *represents the items sold on the visitor side,* C *represents the cost of the items,* H_I *represents the items place in inventory on the home side, and* V_I *represents the items placed in inventory on the visitor side. Students will need to align the matrices and elements so they can perform the matrix operations.*

Indicator: A video store owner has found the probability that a customer who rented a movie today will rent a movie tomorrow is 35%. The probability that a customer who did not rent a movie today, but will rent tomorrow is 10%. Write a transition matrix that represents this information.

Answer: $\begin{bmatrix} .35 & .65 \end{bmatrix}$

Indicator: A population of laboratory animals models the following birth and survival rates. Write the Leslie matrix that would be used to model population growth.

Age	Birthrate	Survival Rate
0-3	0.3	0.9
3-6	0.9	0.7
6-9	0.9	0.8
9-12	0.6	0

Months	0-3	3-6	6-9	9-12
Number	10	11	8	4

Answer:

$$\begin{bmatrix} 0.3 & 0.9 & 0.9 & 0.6 \\ 0.9 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0 \\ 0 & 0 & 0.8 & 0 \end{bmatrix}$$

Return to: Standards

DCS.N.2 Understand matrices to solve problems. DCS.N.2.2 Interpret solutions found using matrix operations including Leslie Models and Markov Chains, in context.	
Clarification	**Checking for Understanding**
Students should be able to interpret the solutions of matrix operations based on the context of the problem. This can include, interpret the matrix as a whole, interpreting elements within the solution, and interpreting any intermediate steps taken in the solving process. For Leslie Models and Markov Chains, students should be able to: ● Organize the data into the appropriate matrices (DCS.N.2.1) and ● Evaluate the matrices (DCS.N.1.1). The expectation of this standard is that students will be able to interpret the results of the Leslie Model and Markov chains.	**Indicator**: Riverside High School Booster Club is running the concession stands at a JV Football game. ● Booster club sells cans of soda for $1, a slice of pizza for $3, a chicken sandwich for $5, and candy for $1.50. ● The home side concession stand sold 160 cans of soda, 82 slices of pizza, 103 chicken sandwiches, and 72 pieces of candy. ● The visitors side concession stand sold 102 cans of soda, 65 slices of pizza, 52 chicken sandwiches, and 37 pieces of candy. ● The booster club purchased the soda for $.25 per can, each slice of pizza for $1.5, each chicken sandwich for $3.29, and each piece of candy for $.70. ● The home side concession was stocked with 200 cans of soda, 96 slices of pizza, 120 chicken sandwiches, and 150 pieces of candy. ● The visitors side concession stand was stocked with 120 cans of soda, 80 slices of pizza, 80 chicken sandwiches, and 100 pieces of candy. a. In the context of this problem, what does $129.29 represent? b. How much money did the booster club spend on stocking pizza for both concessions stands? *Answers: a.* The booster club's profit from the visitors concession. b. $264 **Indicator**: A video store owner has found that the probability that a customer who rented a movie today will rent a movie tomorrow is 35%. The probability that a customer who did not rent today, but will rent tomorrow is 10%. a. If 853 out of his 8745 customers rented a movie on Monday night, how many customers can he expect to rent a movie on Tuesday? b. About how many of his customers can be expected to rent a movie three weeks from Monday and explain how you can know this without calculating each day? *Answer: a. Tuesday will have 1088 renters. b. Three weeks from Monday will have 1166 renters.*

	Indicator: A population of laboratory animals models the following birth and survival rates. The following table displays the initial population: a. How many newborn animals were there after four cycles? b. What is the total population after two cycles? c. When will the population reach 800? d. What is the long-term growth rate? *Answer: a. 47, b. 58, c. During the 11th cycle, d. About 34%*

Age	Birthrate	Survival Rate
0-3	0.3	0.9
3-6	0.9	0.7
6-9	0.9	0.8

Months	0-3	3-6	6-9	9-12
Number	10	11	8	4

Return to: Standards

DCS.N.2 Understand matrices to solve problems.

DCS.N.2.3 Represent a system of equations as a matrix equation.

Clarification	Checking for Understanding
Students will use a matrix equation to represent a system of equations from a context. The number of variables and equations are not limited.	**Indicator**: Represent the following system as a matrix equation. $\begin{cases} 5x - y = 7 \\ 2x + 3y = -1 \end{cases}$ *Answer:* $\begin{bmatrix} 5 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ -1 \end{bmatrix}$ **Indicator**: During a soccer season, referees are paid different rates for different types of games. There are three types of games in a typical season: non-conference, conference, and playoff games. There are two referees for each game and schools only have to pay for home games. The information below is from three high schools with the same pay scale for referees. Write a matrix equation based on the context. *Answer:* $\begin{bmatrix} 5 & 7 & 2 \\ 3 & 8 & 1 \\ 6 & 6 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1806 \\ 1570 \\ 1476 \end{bmatrix}$

High School	Home Games Nonconference	Home Games Conference	Playoff Games	Total Pay for Soccer Referees
Green River	5	7	2	$1806
Blue Creek	3	8	1	$1570
Black Lake	6	6	0	$1476

Return to: Standards

DCS.N.2 Understand matrices to solve problems.

DCS.N.2.4 Use inverse matrices to solve a system of equations with technology.

Clarification	Checking for Understanding
Students will use technology and inverse matrices to solve systems of equations from a context. The number of variables and equations in the system is not limited.	**Indicator**: It is the end of the semester. You and two friends cannot remember how your teacher weighted your grades to determine your final score. You and your friend have received the following information. Determine how much each of the categories are weighted.

Quizzes	Tests	Exam	Final
95	92	91	92.34

Quizzes	Tests	Exam	Final
72	83	86	81.62

Quizzes	Tests	Exam	Final
87	72	65	73.3

Answer: Quizzes - .18, Tests - .62, Exam - .2

Indicator: During a soccer season, referees are paid different rates for different types of games. There are three types of games in a typical season: non-conference, conference, and playoff games. There are two referees for each game and schools only have to pay for home games. The information below is from three high schools with the same pay scale for referees.

High School	Home Games Nonconference	Home Games Conference	Playoff Games	Total Pay for Soccer Referees
Green River	5	7	2	$1806
Blue Creek	3	8	1	$1570
Black Lake	6	6	0	$1476

How much is each referee paid for each type of game?

Answer: Nonconference- $110, Conference - $136, and Playoff - $152

Return to: Standards

DCS.N.3 Understand set theory to solve problems.

DCS.N.3.1 Recognize sets, subsets, and proper subsets.

Clarification	Checking for Understanding
Since middle school students have been informally working with the concept of a set. In this course, students are formalizing the definition and notation of sets, subsets, and proper subsets. Students should be able to interpret set notation to identify subsets and proper subsets. Student should be able to recognize the following set notation symbols: { } *empty set**; $\emptyset$ *null set**; $\in$ *"is an element in"*; $\notin$ *"is not an element in"* $\vert$ *"such that"*; $\mathbb{N}$ *natural numbers*; $\mathbb{Z}$ *integers*; $\mathbb{Q}$ *rational numbers* $\mathbb{R}$ *real numbers*; $\subseteq$ *subset*; $\subset$ *proper subset*; $\cup$ *universal set* ** In set theory, the empty set and null set have the same meaning.*	**Indicator**: Let B = $\{n \in Z \vert n^2 < 20\}$. a. Describe B using the listing method. b. List subsets of {a, b, c}. c. List subsets of {a, b, c, d}. *Answer:* *a. {-4, -3, -2, -1, 0, 1, 2, 3, 4}* *b. $\emptyset$, {a}, {b}, {c}, {a,b}, {b,c}, {a,c}, {a,b,c}* *c. $\emptyset$, {a}, {b}, {c}, {d}, {a,b}, {b,c}, {a,c}, {a,d}, {b,d}, {c,d}, {a,b,c}, {a,b,d}, {a,c,d}, {b,c,d}, {a,b,c,d}*

Return to: Standards

DCS.N.3 Understand set theory to solve problems.

DCS.N.3.2 Implement set operations to find unions, intersections, complements and set differences with multiple sets.

Clarification	Checking for Understanding
Students were introduced to unions, intersections, complements in NC Math 2 in the context of describing a sample space. In this course, students will be asked to work with multiple sets and to find set differences. Student should be able to recognize the following set operation symbols: $\cup$ *union*; $\cap$ *intersection*; A' *complement of set A*; A^c *complement of set A* $A - B$ *set difference, "A cut down by B"*; $A \backslash B$ *set difference, "A cut down by B"* Please note that some set operations have multiple common symbols. Set difference can also be called the relative difference. For example, $A - B$ can be called the relative difference between A and B.	**Formative Check**: Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{2, 4, 6, 8, 10\}$, $B = \{1, 3, 5, 7, 9\}$, and $C = \{1, 2, 3, 4\}$. Describe each of the following sets using the listing method. a. $(A \cup B) \cap C$ b. $A \cap B \cap C$ c. A^c d. $(A \cup C)'$ e. $A \backslash C$ f. $C - B$ *Answer: a. {1,2,3,4}, b. $\emptyset$, c. {1,3,5,7,9}, d. {5,7,9}, e. {6,8,10}, f. {2,4}*

Indicator: In a recent survey, 278 people were asked if they had visited 3 popular restaurants: Macburger (MB), Pizza House (PH), and Dairy Blast (DB). Below are the survey results.

MB	PH	DB	MB & PH	MB & DB	PH & DB	MB, PH, & DB
112	103	94	39	34	40	18

a. How many people did not visit any of the restaurants?
b. How many people only visited Dairy Blast?
c. Explain where in this problem you see examples of unions, intersections, complements and set differences.

Answer: a. 64, b. 38, c. sample answers, MB & PH is an example of an intersection, visited only DB is set difference of the union of MB & PH

Return to: Standards

DCS.N.3 Understand set theory to solve problems.

DCS.N.3.3 Represent properties and relationships among sets using Venn diagrams.

Clarification

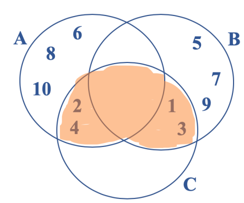
Students use Venn diagrams to represent the relationships between sets. Venn diagrams are often used to visually represent set operations.
Students can use Venn Diagrams to explore the properties of set operations.
Students should know where to represent all of the elements in the universal set.

Checking for Understanding

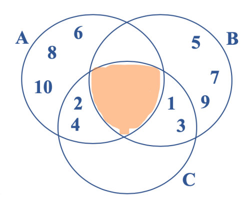
Indicator: Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{2, 4, 6, 8, 10\}$, $B = \{1, 3, 5, 7, 9\}$, and $C = \{1, 2, 3, 4\}$.
Describe each of the following set operations using Venn diagrams.

a. $(A \cup B) \cap C$ b. $A \cap B \cap C$ c. A^c
d. $(A \cup C)'$ e. $A \setminus C$ f. $C - B$

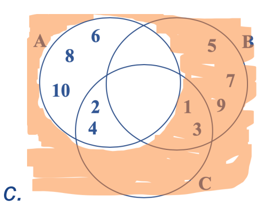
Answer:



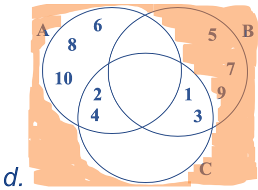
a.

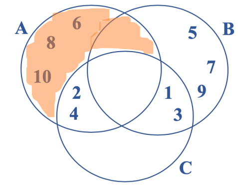


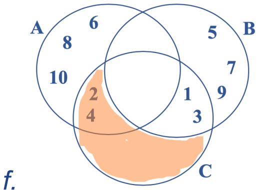
b.



c.

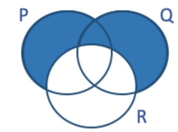
d. 

e. 

f. 

Indicator: Create and shade the portion of the Venn diagram that illustrates the following set: $(P \cup Q) \cap R^C$

Answer:

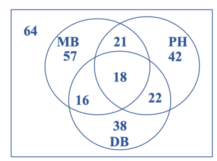


Indicator: In a recent survey, 278 people were asked if they had visited 3 popular restaurants: Macburger (MB), Pizza House (PH), and Dairy Blast (DB). Below are the survey results.

MB	PH	DB	MB & PH	MB & DB	PH & DB	MB, PH, & DB
112	103	94	39	34	40	18

Draw a Venn Diagram to represent this situation. Find any information not given.

Answer:



Return to: Standards

DCS.N.3 Understand set theory to solve problems. DCS.N.3.4 Interpret Venn diagrams to solve problems.	
Clarification	**Checking for Understanding**
Venn diagrams can be an important tool to organize data from sets. This organizational tool can assist in solving problems.	**Indicator**: In a recent survey, 278 people were asked if they had visited 3 popular restaurants: Macburger (MB), Pizza House (PH), and Dairy Blast (DB). Below are the survey results.

MB	PH	DB	MB & PH	MB & DB	PH & DB	MB, PH, & DB
112	103	94	39	34	40	18

Using a Venn Diagram, answer the following questions.

a. How many people did not visit any of the restaurants?
b. How many people only visited Dairy Blast?
c. How many people visited Macburger and Pizza House but not Dairy Blast?

Answer: a. 64, b. 38, c. 21

Return to: Standards

DCS.N.4 Understand statements related to number theory and set theory. DCS.N.4.1 Use the Euclidean Algorithm to determine the greatest common factor and least common multiple.	
Clarification	**Checking for Understanding**
Students were taught GCF and LCM, using prime factorization and other methods in 6th grade. In middle school, students relied on their intuition and experience to guide the process. In this standard, students will learn an iterative process, the Euclidean Algorithm, to find the GCF and LCM. This can serve as a point of comparison between human intuition and iterative processes needed in computer science and programming. Students should have an understanding of the mathematical reasoning behind the Euclidean Algorithm and finding the LCM. (See DCS.N.4.4) <u>Euclidean Algorithm</u> 1. Find the gcf of *a* and *b*, letting *a* be the largest number. Notated as gcf(*a*,*b*). The largest possible factor of *a* and *b*, would be *b*. If $(a) \div b$ has a remainder of zero, *b* is the gcf. If	**Indicator**: Use the Euclidean algorithm to find the greatest common factor of 420 and 130. *Answer:* 1. *gcf (420, 130):* $420 \div 130 = 3\,r30$ 2. *gcf (130, 30):* $130 \div 30 = 4\,r10$ 3. *gcf (30, 10):* $30 \div 10 = 3\,r0$ *Since 10 divides 30, then gcf(420,130)=10.* **Indicator**: Use the Euclidean algorithm to find the least common multiple of 420 and 130. *Answer: Since* $gcf(420, 130) = 10$*, then* $lcm(420, 130) = 420 \cdot 130 \div 10 = 5460$

the remainder of $(a) \div b$ is not zero, then we know that $a = n \cdot b + r_1$, where *n* is an integer and r_1 is the remainder. 2. If $b \div r_1$ has a remainder of zero, r_1 is the gcf. If the remainder is not zero, then we know that $b = n \cdot r_1 + r_2$, where *n* is a number and r_2 is the remainder. 3. If $r_1 \div r_2$ has a remainder of zero, r_2 is the gcf. If the remainder is not zero, then we know that $r_1 = n \cdot r_2 + r_3$, where n is a number and r_3 is the remainder. 4. Repeat this pattern until the remainder is 0. The divisor of the expression that produced the remainder of 0 is the gcf(*a*,*b*) Using the Euclidean Algorithm to find the GCF, the LMC a and b can be found using the gcf(a,b). $lcm(a, b) = \frac{a \cdot b}{gcf(a,b)}$ Students should be able to recognize other mathematical terms that should also be interpreted as GCF or LCM, such as greatest common divisor or least common denominator.	

Return to: Standards

DCS.N.4 Understand statements related to number theory and set theory. DCS.N.4.2 Use the Fundamental Theorem of Arithmetic to solve problems.	
Clarification	**Checking for Understanding**
Students began working with this concept when writing the prime factorization of a number, and found the GCF and LCM in 6th grade. In this course, students will use their knowledge of the Fundamental Theorem of Arithmetic to solve various problems. Some of these problems may include: ● Multiplication and Division ● Determining if a number is: odd, even, prime, composite, perfect square, perfect cube Students should have an understanding of these relationships. (See DCS.N.4.4)	**Indicator**: Use the Fundamental Theorem of Arithmetic to find the least positive integer n such that $(2^5)(3)(5^2)(7^5)(n)$ is a perfect square. Write the resulting product as a perfect square. Explain. *Answer: To be a perfect square, all of the exponents must be divisible by 2.* $n = 2 \cdot 3 \cdot 7 = 42 - n$*, must contain the "missing" prime factors necessary to make the exponents divisible by 2.* $(2^5)(3)(5^2)(7^5)(42)$ *- substitute 42 for* n $(2^6)(3^2)(5^2)(7^6)$ - rewrite the expression as a product of prime factors $((2^3)(3^1)(5^1)(7^3))^2$- rewrite the expression as a perfect square $41,160^2$- evaluate the product of prime factors

Return to: Standards

DCS.N.4 Understand statements related to number theory and set theory.

DCS.N.4.3 Conclude that sets are equal using the properties of set operations.

Clarification

Students may use a variety of tools to determine the equality of sets, including the listing method, Venn diagram, and logical statements. Sets are equivalent when the sets contain the same elements.
Students are expected to know the basic properties of set operations for unions and intersections.

Properties	**Unions**	**Intersections**
Commutative	If $A \cup B$, then $B \cup A$.	If $A \cap B$, then $B \cap A$.
Associative	If $(A \cup B) \cup C$, then $A \cup (B \cup C)$.	If $(A \cap B) \cap C$, then $A \cap (B \cap C)$.
Distributive	If $A \cup (B \cap C)$, then $(A \cup B) \cap (A \cup C)$.	If $A \cap (B \cup C)$, then $(A \cap B) \cup (A \cap C)$.
Empty Set	$A \cup \emptyset = A$	$A \cap \emptyset = \emptyset$

Checking for Understanding

Indicator: Make a conjecture about the following:
a. the number of subsets of a set with five elements.
b. the number of subsets of a set with n elements.

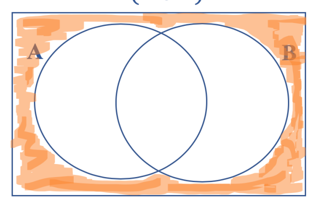
Answer: a. 2^5 subsets, b. 2^n subsets

Indicator: Provide a justification for the following:

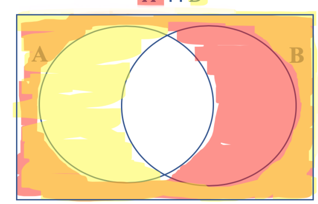
If A and B are sets, then $(A \cup B)' = A' \cap B'$

Answer: Using Venn Diagrams to show equivalence.

$(A \cup B)'$



$A' \cap B'$



Return to: Standards

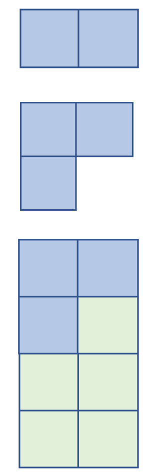
<table>
<tr><td colspan="2">DCS.N.4 Understand statements related to number theory and set theory.

DCS.N.4.4 Explain theorems related to greatest common factor, least common multiple, even numbers, odd numbers, prime numbers, and composite numbers.</td></tr>
<tr><td>Clarification</td><td>Checking for Understanding</td></tr>
<tr><td>The expectation of this standard is that students will be able to informally explain the mathematical reasoning behind theorems related to the topics listed and how they relate to a problem being solved. For example, students should not only be able to explain the Fundamental Theorem of Arithmetic, but would also be able to explain how the student uses the theorem to solve a problem.

The theorems targeted for students' explanations should be those that are used in other standards throughout this course, such as those seen in:
● DCS.N.4.1 - GCF, LCM
● DCS.N.4.2 - Even, odd, prime, and composite numbers</td><td>Indicator: Suppose x is an odd integer and y is an odd integer. Is $x + y$ an even or odd integer? Provide a justification.
Answer: Sample answers below
Algebraic approach
If $2n$ is an even number, then $2n+1$ is an odd number.
$(2n_1 + 1) + (2n_2 + 1)$, adding two odd numbers
$2n_1 + 2n_2 + 2$, rewriting using the properties of operations
$2(n_1 + n_2 + 1)$, rewriting the expression by factoring a two.

This shows that the sum of two odd numbers is a multiple of 2, which is the definition of an even number.

Drawing Approach
If a number is represented as objects in two columns, an even number will always have a complete row and an odd number will always have one row with only one object. (See to the right.)

When adding two odd numbers, each number has one row with one object. As the numbers are added, the two rows with one object will be combined to make a complete row. With all rows having two objects, the sum represents an even number.
</td></tr>
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Return to: Standards

DCS.F.1 Apply recursively defined relationships to solve problems.

DCS.F.1.1 Implement procedures to find the *nth* term in an arithmetic or geometric sequence using spreadsheets.

Clarification

This mathematical content of this standard has been covered in previous high school math courses.
The expectation of this standard is that students solve arithmetic and geometric problems using a spreadsheet. The spreadsheet application used is at the discretion of the instructor. For some students, this may require instruction on the use of the chosen spreadsheet application.

Checking for Understanding

Formative Check: Given two terms in a geometric sequence, find the 8th term and the recursion formula.

$$a_5 = 768 \text{ and } a_2 = 12$$

Answers: $a_8 = 49,125,\ a_n = 4 \cdot a_{n-1}$

Indicator: Given a term in an arithmetic sequence and the common difference, find the recursive formula and the three terms in the sequence after the last one given.

$$a_{22} =- 44 \text{ and } d =- 2$$

Answers: $a_{23} =- 46,\ a_{24} =- 48,\ a_{25} =- 50,\ a_n = a_{n-1} - 2$

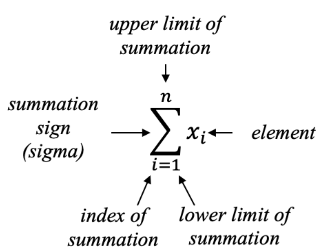
Return to: [Standards]

DCS.F.1 Apply recursively defined relationships to solve problems.

DCS.F.1.2 Represent the sum of a sequence using sigma notation.

Clarification

Students should be able to write a summation using sigma notation.



Students should be able to distinguish when the index will be used directly in the calculation of the sum and when the index is a reference to an element of a set.

Checking for Understanding

Indicator: Use sigma notation to represent the sum of the first 10 squares, starting with 1.

Answer: $\sum_{x=1}^{10} x^2$

Indicator: Use sigma notation to represent the 7 elements of a series. The elements of the series are the difference of three times an element of set a and 1.

Answer: $\sum_{i=1}^{7} (3a_i - 1)$

Return to: [Standards]

DCS.F.1 Apply recursively defined relationships to solve problems. DCS.F.1.3 Implement procedures to find the sum of a finite sequence.	
Clarification	**Checking for Understanding**
Students are expected to find the sum of a finite sequence. Students should be able to distinguish between a sequence and a series. For this standard, students may be asked to find the sum of a sequence or series. Students should be able to interpret the use of ellipsis, …, in a mathematical context. Students may use the formulas for sum of finite arithmetic and geometric sequences.	**Formative Check**: Find the sum of the given geometric or arithmetic sequence or series. a. $\sum_{n=1}^{11} 4(-\frac{1}{3})^{n-1}$ b. $5, 15, 45, ..., 98415$ c. $b = \{3, 9, 12, 21\}$ $\sum_{i=2}^{3} \frac{1}{3} b_i^2$ *Answer: a. about 3, b. 147,620, c. 75* **Indicator**: Sarah puts $300 in a savings account that earns 5.25% compounded annually. a. Write a recursive relation for the situation. b. During what year will Sarah double her money? *Answer: a.* $a_n = 1.0525 \cdot a_{n-1}$, *b. During the middle of 13th year.*

Return to: Standards

DCS.F.1 Apply recursively defined relationships to solve problems. DCS.F.1.4 Implement procedures to find the sum of an infinite sequence and determine if the series converges or diverges.	
Clarification	**Checking for Understanding**
For this standard, students may be asked to find the sum of a sequence or series. Students should be able to interpret the use of ellipsis, …, in a mathematical context. Students should be able to use partial sums to verify whether a sequence is convergent or divergent. If the sequence converges, students should be able to hypothesize the value the sum approaches. Students may use the formula for the sum of infinite geometric sequences.	**Formative Check**: Find the sum of the series. a. $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1}$ b. $5 + 20 + 80 + 320 + 1280 +..$ *Answer: a. 2, b. ∞* **Indicator**: Explain how you know if the following series converges or diverges. If it does converge, find the sum. $\sum_{n=1}^{\infty} 3(.75)^{n-1}$ *Answer: Since* $\lvert r \rvert < 1$, *each successive element will be smaller than the*

	previous until at a point at which the magnitude of the element will no longer change the sum. This series will converge at 12. **Indicator**: A tennis ball is launched from the ground. It travels a total of 47 feet from the launch to the first contact with the ground. In each successive bounce, the tennis ball will travel 53% of the previous bounce. How far will the tennis ball travel in total? *Answer: 100 ft*

Return to: Standards

DCS.F.1 Apply recursively defined relationships to solve problems. DCS.F.1.5 Interpret the solutions to arithmetic and geometric sequences and series problems, in context.	
Clarification	**Checking for Understanding**
Students should be able to describe the solution to problems involving sequence and series problems using the context of the problem.	**Indicator**: An auditorium has 18 seats in the first row. Each successive row has two additional seats. The last row has 84 seats. a. Write a relation for the number of seats in each row. b. What information can you obtain from that relation? *Answer: a.* $s_r = s_{r-1} + 2$ *or* $s = 2(r - 1) + 18$*, b. the number of seats in any row, the total number of seats or rows in the auditorium* **Indicator**: A tennis ball and super ball are launched from the ground. Each ball travels to a height of 100 ft. The height a tennis ball travels during each successive bounce is reduced by 47%. The height a super ball travels during each successive bounce is reduced by 15%. Jon is 6 ft tall. After how many bounces will each ball's height be less than Jon's height? *Answer: Tennis ball - 5 bounces, super ball - 18 bounces*

Return to: Standards

DCS.SP.1 Apply combinatorics concepts to solve problems. DCS.SP.1.1 Implement the Fundamental Counting Principle to solve problems.	
Clarification	**Checking for Understanding**
The foundation for this standard was laid in 7th grade, during the exploration of the probability of compound events (NC.7.SP.8). Students should be able to use the Fundamental Counting Principle, also called Multiplication Principal, to solve problems in a context.	**Indicator**: You take a survey with five "yes" or "no" answers. How many different ways could you complete the survey? *Answer: 2 * 2 * 2 * 2 * 2 = 32.* **Indicator**: For the school lunch line, each student has a choice between 2 main dishes, 3 side dishes and 2 drinks. If a student must choose one of each, how many combinations of the lunch are possible? *Answer: 12 combinations are possible*

Return to: Standards

DCS.SP.1 Apply combinatorics concepts to solve problems. DCS.SP.1.2 Implement procedures to calculate a permutation or combination.	
Clarification	**Checking for Understanding**
Students should be able to use permutations and combinations to solve problems in a context. The number of different permutation or combinations possible can be found using the formulas, ${}_nP_k = \frac{n!}{(n-k)!}$ or ${}_nC_k = \frac{n!}{k!(n-k)!}$, with n being the objects available with k objects being selected at a time. In this course, the focus should remain on properly interpreting the context to determine the correct procedure or combination of procedures needed to find the solution.	**Indicator**: In how many ways can a group of 5 members be formed by selecting 3 boys out of 6 and 2 girls out of 5? *Answer:* $C(6, 3)\cdot C(5, 2) = 20\cdot 10 = 200$ **Indicator**: How many more ways can 10 juniors running for the positions of president, vice president, secretary, and treasurer be selected when compared to 12 sophomores running for 5 identical positions of class representative? *Answer:* $P(10, 4) - C(12, 5) = 4248$

Return to: Standards

DCS.GT.1 Understand graph theory to model relationships and solve problems.

DCS.GT.1.1 Represent real world situations using a vertex-edge graph, adjacency matrix, and vertex-edge table.

Clarification

Students should be able to create vertex-edge graphs, adjacency matrices, and vertex-edge tables from a context.

A vertex-edge graph consists of points (vertices) and connections between the points (edges) that represent a context.
Students may be asked to create several types of vertex-edge graphs, including a tournament graph. See GT.1.3 for more information and interpreting tournament graphs.

Two vertices are considered adjacent if there exists an edge between them. In an adjacency matrix, adjacent vertices are represented with a 1. No adjacent vertices are represented with a 0.

A vertex-edge table consists of a table which displays the number of edges from each vertex. The number of edges from a vertex is also known as the degree of that vertex. Vertex-edge tables can include listing the adjacent vertices of each vertex. In other sources, this may be known as an edge table.

Checking for Understanding

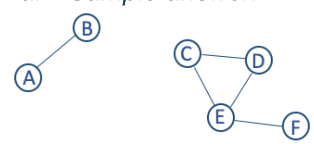
Indicator: The table shows a list of students who are Facebook "friends."

a. Create a vertex edge graph that represents the information in the table.
b. Construct an adjacency matrix to represent the information in the table.

Student	Is friends with:
Anna	Beth
Chun	Dalton
Dalton	Edwin
Edwin	Chun
Edwin	Felicia

Answers:

a. Sample answer.



b.

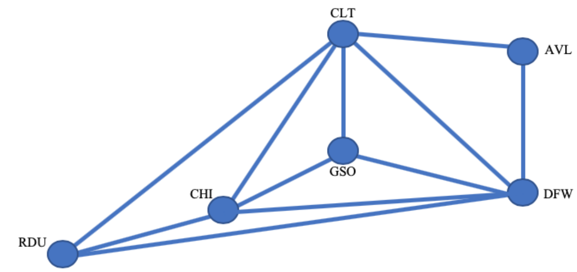
$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Indicator: Use the airport abbreviations for the following cities:
Asheville (AVL), Charlotte (CLT), Chicago (CHI), Dallas (DFW), Greensboro (GSO), and Raleigh (RDU)
American Airlines has direct flights between the following airports:
CLT & AVL, CLT & DFW, CLT & GSO, CLT & CHI, CLT & RDU, AVL & DFW, GSO & DFW, GSO & CHI, RDU & DFW, RDU & CHI, CHI & DFW

a. Create a vertex-edge graph to represent the direct flights between the cities.
b. Create an adjacency matrix to represent the direct flights between the cities.
c. Create a vertex-edge table to represent the direct flights between the cities.

Answers:

a.



b.

$$\begin{array}{c|c} & \begin{array}{cccccc} \text{AVL} & \text{CLT} & \text{CHI} & \text{DFW} & \text{GSO} & \text{RDU} \end{array} \\ \begin{array}{c} \text{AVL} \\ \text{CLT} \\ \text{CHI} \\ \text{DFW} \\ \text{GSO} \\ \text{RDU} \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

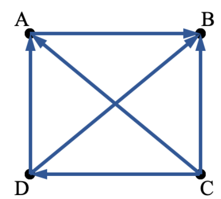
c.

Vertex	Edges
AVL	2
CLT	5
CHI	4
DFW	5
GSO	3
RDU	3

Indicator: Create a tournament graph to represent the round robin tournament represented below:

Game	AB	AC	AD	BC	BD	CD
Winner	A	C	D	C	D	C

Answer:



Return to: Standards

DCS.GT.1 Understand graph theory to model relationships and solve problems.

DCS.GT.1.2 Test graphs and digraphs for Euler paths, Euler circuits, Hamiltonian paths, or Hamiltonian circuits.

Clarification

Students evaluate graphs and digraphs to determine if they meet the criteria for being an Euler path, Euler circuit, Hamiltonian path, or Hamiltonian circuit. A digraph is a directed graph in which the flow is from one vertex to another vertex represented by arrows.

Euler Circuits and Paths

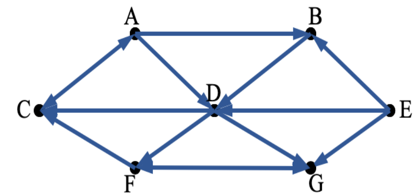
While Euler circuits and paths can be found through trying random paths, as the graphs and digraphs become more complex, testing can become cumbersome. Students should know how to use the degree of the vertices to make a quick determination. The degree of a vertex is the number of edges that have the vertex as an endpoint. To be an Euler circuit, the degree of all vertices of a connected graph must be even. To be an Euler path, the connected graph has exactly two vertices with an odd degree.

Hamiltonian Circuits and Paths

Unlike Euler paths and circuits, there are no simple tests to determine if a graph or digraph is a Hamiltonian circuit or path. Some theorems do exist for certain categories of graphs. Knowing and using those theorems is not an expectation of this course.
Understanding a Hamiltonian circuit is essential to Traveling Salesperson Problems which are addressed in GT.2.2.

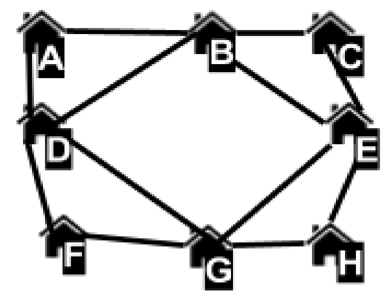
Checking for Understanding

Indicator: A group of 7 Discrete math students are working on a project and plan to share information via text messages. The digraph below shows the 7 students' communication outside of class. Does this represent a Hamiltonian circuit? Explain.



Answers: No, a Hamiltonian Circuit does not exist as all directions are away from E, so even if the path started at E, it could never return to E.

Indicator: The graph below shows a small neighborhood of homes along with roads connecting them.



a. Is it possible for the person in house B to travel to each house to hand out flyers for her missing pet pig and return back home without passing by the same house twice? If so, name the circuit, and if not, explain.
b. The latest snow storm has dumped over a foot of snow. Is it possible for the person in House F to plow every road exactly once without traveling over the same road twice? If so, name the circuit and if not, explain.

Answers:

a. Yes, it is possible. One example is B-C-E-H-G-F-D-A-B
b. Yes, it is possible since every vertex has an even degree. One example is F-D B-E-G-D-A B-C-E H-G-F

Indicator: Create a tournament graph to represent the round robin tournament below and list all possible Hamiltonian paths.

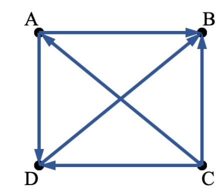
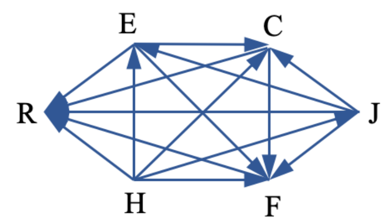
Game	AB	AC	AD	BC	BD	CD
Winner	A	C	D	C	D	C

Answer: See GT1.1 for the tournament graph. There is only one possible path, C - D - A - B

Return to: Standards

DCS.GT.1 Understand graph theory to model relationships and solve problems.

DCS.GT.1.3 Interpret a complete digraph to determine rank.

Clarification	Checking for Understanding
In this objective, students examine a tournament graph to determine the ranking of participants in a tournament. A tournament graph is a directed complete graph in which a result of the interaction is represented by the direction of the arrow. The base of the arrow starts with the "winner" and points to the "loser." In a complete graph, all vertices are connected to all other vertices by an edge. The context of a tournament graph does not have to be limited to a tournament. From the tournament graph, students should be able to interpret the ranking of the competitors in relation to each other. Students may be asked to create a tournament graph to reflect rankings (DCS.GT.1.1). *Note: It is not the expectation of this standard for students to determine the rank of a digraph. The rank of a digraph is a distinct and more complex concept.*	**Indicator**: The graph represents the results of a round-robin tournament held by the robotics club at North High School. Determine the final rankings for teams A, B, C, and D. *Answer: Highest to lowest: C - A - D - B*  **Indicator**: The graph represents the bowling championship tournament for the following high school teams: Riverdale, Edwin, Johnson, Curry, Hamilton, and Franklin.  a. Who was the champion? b. How many wins did each team achieve? *Answer: a. Hamilton, b. Riverdale 0, Edwin 3, Johnson 4, Curry 2, Hamilton 5, and Franklin 1.*

Return to: Standards

DCS.GT.2 Apply graph theory to solve problems.

DCS.GT.2.1 Implement critical path analysis algorithms to determine the minimum project time.

Clarification

Students are expected to complete a critical path analysis to find the minimum project time. The critical path of a project is the critical path from start to end. Processing time is the sum of the time to complete all steps in a path. For the critical path, the processing time is also known as the critical time or the minimum project time.

A systematic method to find the critical path is using the <u>Backflow Algorithm</u>.
In this method:
1. Beginning at the End, work backward to find the critical time, the longest total processing time, for each vertex.
2. Record the critical time for each vertex.
3. Find and highlight the path that creates the critical time from the Start.

Note: How the processing time and critical time are represented in a graph can vary.

<u>The Critical Path Algorithm</u>
1. Find the critical times using the backflow algorithm.
2. Create a critical-time priority list critical times.
3. Create schedule

Only the first step is required to find the minimum project time, as the minimum project time is the critical time from the Start.

The other steps in the Critical Path Algorithm can be added as time allows or as an extension.

Students can be asked to find the minimum project time of an already completed schedule, as long as all prerequisite information is given.

Note: Names and methods of representation critical paths can vary depending on the source.

Checking for Understanding

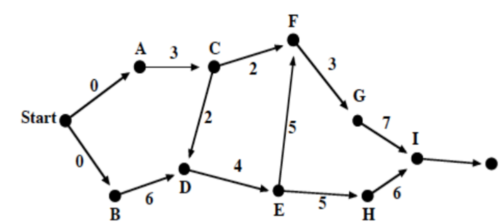
Indicator: Create a graph to represent the project listed.

a. What is the earliest start time for Task G?
b. What is the latest start time for Task B?
c. Name the critical path.
d. What is the minimum project time required?

Answers: a.13, b.1, c. Start-A-C-D-E-G-Finish, d.14

Task	Time	Prerequisite
A	3	none
B	4	none
C	2	A
D	3	B, C
E	5	D
F	2	B
G	1	E,F

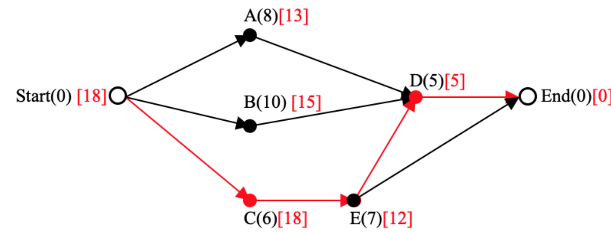
Indicator: According to the graph shown below:



a. How many prerequisites does F have?
b. What is the earliest start time for E?
c. What is the minimum project time?
d. What is the critical path?

Answers: a. 5, b. 9, c. 24, d. Start - B - D - E - F - G - I - End

Indicator: A backflow algorithm has been applied to a potential project and is shown in the graph below. The units are in hours.



a. What is the processing time and the critical time for task E?
b. How was the critical time for E calculated?
c. What is the minimum project time for this project?
d. What does the minimum project time tell us?

Answers: a. The processing time is 7 hours and the critical time is 12 hours. b. The critical time was calculated using the longest total time from E to the End. This goes from E-D-End, which is 7+5=12 hrs. c.18 hrs d. The minimum project time tells us the minimum amount of time possible to complete all tasks in the project.

Return to: [Standards]

DCS.GT.2 Apply graph theory to solve problems.

DCS.GT.2.2 Implement the brute force method, the nearest-neighbor algorithm, and the cheapest-link algorithm to find solutions to a Traveling Salesperson Problem.

Clarification

Traveling Salesperson Problems (TSP) are a common category of problem in which the goal is to determine the most efficient pathway. TPS problems involve a variety of context other than a traveling salesperson.
Students should know that all Traveling Salesperson problems involve Hamiltonian circuits.

<u>Brute Force method</u>:

1. List all Hamiltonian circuits.
2. Calculate the total weight of each circuit.
3. Choose an optimal circuit.

Students should know that the brute force method produces the most optimal path. However, the time to complete the brute force method increases greatly for each additional vertex.

<u>Nearest-Neighbor algorithm</u>:

1. From a starting vertex choose the edge with the lowest edge weight (nearest neighbor).
2. From that vertex chose the next edge with the lowest weight to go to the next vertex and continue in this pattern.
3. From the last vertex, travel back to the starting vertex.

In this course, the starting vertex will be provided when students are asked to complete this algorithm. The repetitive nearest-neighbor algorithm is not an expectation of this standard.

Checking for Understanding

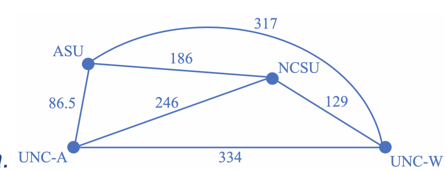
Indicator: Kayana decided to do a driving tour of 4 college campuses to determine which school(s) she is most interested in attending. The table below shows the distances between each university.

	ASU	NCSU	UNCA	UNCW
ASU	--------	186	86.5	317
NCSU	186	--------	246	129
UNCA	86.5	246	--------	334
UNCW	317	129	334	--------

a. Draw a weighted graph using this data.
b. What is the total distance Kayana will travel using the Nearest Neighbor Algorithm, if she begins and ends her travel at Appalachian State University (ASU)?
c. What is the total distance Kayana will travel using the Cheapest Link algorithm, if she begins and ends at ASU?
d. What is the optimal solution? Draw a tree diagram to show all possible routes.

Answers:

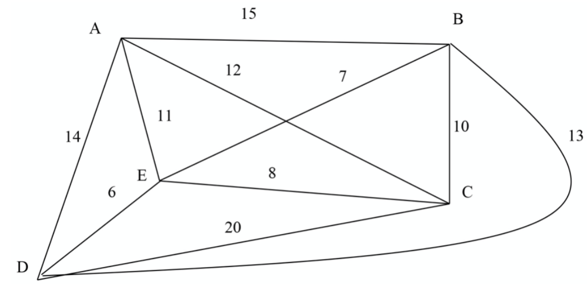
a.



b. (ASU-UNCA)+(UNCA-NCSU)+(NCSU-UNCW)+(UNCW-ASU)
86.5+246+129+317
778.5 miles

c. (ASU-NCSU)+(NCSU-UNCW)+(UNCW-UNCA)+(UNCA-ASU)
186+129+334+86.5
735.5 miles

d. The optimal path forms a quadrilateral.
(ASU-UNCA)+(UNCA-UNCW)+(UNCW-NCSU)+(NCSU-ASU)
86.5+334+129+186
735.5 miles

<u>Cheapest-Link algorithm</u>:
1. Pick the edge with the smallest weight (cheapest) and mark it.
2. Pick the next edge with the smallest weight and mark it.
3. Continue in the process of marking the next smallest weight edge, skipping any edge that would either:
    a. close the circuit or
    b. create any vertex with three edges coming out of a single vertex.
4. Connect the last vertices to close the circuit.
5. If needed, reorder the path to start from a particular vertex based on context.
 	
Other methods and algorithms are not an expectation of this course.
	Indicator: Mr. Deal sells horse feed and must travel to four farms today and then return home. Since he pays for his own gas, he wants to ensure that the distance he travels in total will be the least possible distance.
	a. If his home is at **A**, find the best way for him to travel using the Brute Force Method and then the Nearest Neighbor Method.
	b. Which method is always guaranteed to give you the shortest route? Why is it not always used?
	
	
	Answers:
	a. Brute force:
	A-D-E-B-C-A, 49 units
	Nearest Neighbor:
	A-E-D-B-C-A, 52 units
	b. The Brute force method guarantees the shortest route because it looks at all possibilities. This also makes it an inefficient method.

Return to: [Standards]

DCS.GT.2 Apply graph theory to solve problems.

DCS.GT.2.3 Implement vertex-coloring techniques to solve problems.

Clarification

Vertex coloring is a way of labeling vertices to solve problems involving constraints.

The goal of vertex coloring is to find the minimum number of colors needed. This number is known as the chromatic number.

The events represented by the same color vertex can occur at the same time, as there are no constraints between those events. This makes vertex coloring useful when creating schedules and maps, clustering, data mining, networking, and determining resource allocation.

In this course, vertex coloring problems should remain simple enough to be completed by hand, without the use of advanced algorithms.

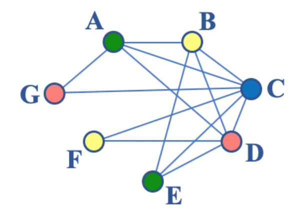
Checking for Understanding

Indicator: It is time for Mrs. Blevins to create a testing schedule for Discrete High School. The difficulty is that some students have multiple tested subjects, so they must be scheduled for different days. The table shows which tests must be scheduled separately.

Test	Cannot be tested with
A	B, C, D, G
B	A, C, D, E
C	A, B, D, G, E
D	A, B, C, E, F
E	B, C, D
F	C, D
G	A, C

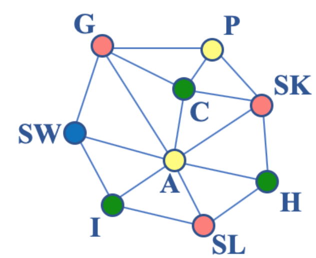
Draw a vertex-edge graph to represent this situation and find the chromatic number. What does the chromatic number mean in this situation?

Answer:The chromatic number is 4, which means Mrs. Blevins must create 4 different testing schedules. Sample graph provided.



Indicator: Represent the following map as a vertex-edge graph. Use the graph to determine the chromatic number.

Answer: Chromatic Number = 4. Sample graph provided.



Return to: [Standards]

DCS.GT.2 Apply graph theory to solve problems.

DCS.GT.2.4 Implement Kruskal and Prim's algorithms to determine the weight of the minimum spanning tree of a connected graph.

Clarification

For this objective, students will determine the weight of a minimum spanning tree.

<u>Kruskal's algorithm</u>:
- Ensure the graph is a spanning tree
- List the edges in order from shortest to longest, breaking ties arbitrarily
- Mark the first edge on the list
- Mark the next edge on the list, unless it would form cycle
- Repeat the last step until n-1 edges have been marked.

The marked edges and vertices are the minimum spanning tree and that weight can be calculated.

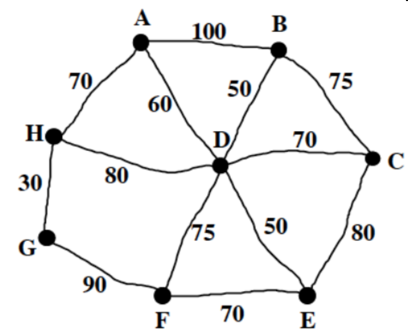
<u>Prim's algorithm</u>:
- Starting with the edge with the least weight, mark that edge and circle the vertices. Ties are broken arbitrarily.
- Find the edge with the least weight that contains one circled vertex and one uncircled vertex. Mark that line and circle the uncircled vertex.
- Repeat the previous step until all vertices are circled.

The marked edges and vertices are the minimum spanning tree and that weight can be calculated.

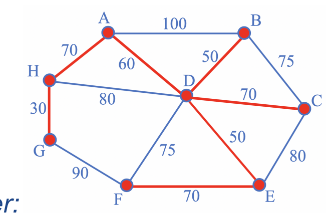
Students should know that the algorithms can produce different minimum spanning trees. Other algorithms are not an expectation of this course.

Checking for Understanding

Formative Check: Use Kruskal's algorithm to find the minimum spanning tree.



Answer:



Indicator: Project: Distribution Centers
Using a map of North Carolina:
- Find and label 8 cities from the various regions of the state
- Create and label a vertex-edge graph that shows the distance needed to travel between your cities. At most, you can connect one city to four other cities.
- All of the cities will have a distribution center and be connected in a distribution network.
- Your goal is to:
  - Identify the most efficient distribution network,
  - Determine the city where the items being distributed will be produced,
  - Determine how many drivers will be needed at each distribution center.
- Restrictions for distribution centers and drivers:
  - Due to travel restriction agreements with the drivers, a driver can only connect to one adjacent city/distribution center.
  - Mileage is the key factor in determining a cost efficient network for your company.
  - It is also more efficient for your company to have as

	many drivers as possible live in the same city.  *Answer: Student answers can vary. In creating an efficient distribution network, the students should determine that they need to find the minimum spanning tree. The method used is at the discretion of the students. The city with the most connections in the minimum spanning tree (MST) is probably the most efficient city to produce the item. The number of drivers can be organized in various ways, keeping in line with the restriction listed in the problem.*

Return to: Standards

DCS.L.1 Evaluate mathematical logic to model and solve problems.

DCS.L.1.1 Construct truth tables that encode the truth and falsity of two or more statements.

Clarification

In this course, students are expected to construct truth tables. In other objectives, students will use truth tables as a tool to solve problems or justify claims.

Student should be able to interpret and use the following logic symbols:

$\neg p$ *the negation of p* *"not p"*	$\sim p$ *the negation of p* *"not p"*	$p \wedge q$ *conjunction of p and q* *"p and q"*
$p \vee q$ *disjunction of p and q* *"p or q"*	$p \oplus q$ *exclusive disjunction of p or q* *"p XOR q"*	$p \rightarrow q$ *p implies q* *"If p then q"*

For this course, the problems will be limited to a total of 3 arguments (premises).

Checking for Understanding

Indicator: Construct a truth table for the statement:

$$\sim P \wedge (P \rightarrow Q)$$

Answer:

P	Q	$\sim P$	$(P \rightarrow Q)$	$\sim P \wedge (P \rightarrow Q)$
T	T	F	T	F
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Indicator: Construct a truth table for the following scenario. You are told that if you are passing your classes, finished your chores, and did not get grounded, then you can go out with your friends for the weekend.

Answer: Sample answer. Let P be for passing grades, C for completed chores, and G for being grounded.

$(P \wedge C) \wedge \sim T$

P	C	$P \wedge C$		$P \wedge C$	G	$\sim G$	$(P \wedge C) \wedge \sim G$
T	T	T		T	T	F	F
T	F	F		T	F	T	T
F	T	F		F	T	F	F
F	F	F		F	F	T	F
				F	T	F	F
				F	F	T	F
				F	T	F	F
				F	F	T	F

Return to: Standards

DCS.L.1 Evaluate mathematical logic to model and solve problems. DCS.L.1.2 Critique logic arguments (e.g. determine if a statement is valid or whether an argument is a tautology or contradiction).	
Clarification	**Checking for Understanding**
In critiquing logical arguments, students should be able to determine the truthfulness of an argument and any claims made in the justification of that argument. Students should be able to suggest corrections, connections, or next steps as needed. As part of determining the truth of an argument, students should be familiar with: ● the law of syllogism, ● the law of detachment, ● tautologies, assertions that are true in all interpretations, and ● contradictions, assertions that are incompatible or incongruous. To evaluate logic statements, students should also be able to understand and use the following logic terms: ● converse ● inverse ● contrapositive ● "if and only if"	**Indicator**: Determine the validity of the following statement: If you invest in Corporation X, then you get rich. You didn't invest in Corporation X. Therefore, you didn't get rich. *Answer:* ● *Let p be the statement "You invest in Corporation X."* ● *Let q be the statement "You get rich."* ● *Then the argument has this symbolic form:* $\begin{array}{c} p \to q \\ \underline{\sim p} \\ \therefore \sim q \end{array}$ ● *Interpret the truth table.*

p	q	premise $p \to q$	premise $\sim p$	conclusion $\sim q$
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Notice that in the third row, the conclusion is FALSE while both premises are TRUE. This tells us that the argument is INVALID.

Indicator: Show that the following logical argument is a tautology.

$$(P \to Q) \vee (Q \to P)$$

Answer:

P	Q	$P \to Q$	$Q \to P$	$(P \to Q) \vee (Q \to P)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

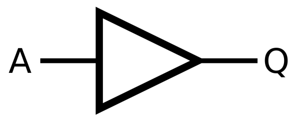
Return to: Standards

DCS.L.1 Evaluate mathematical logic to model and solve problems.

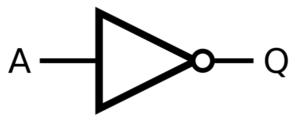
DCS.L.1.3 Check 1s and 0s to determine whether a statement is true or false using Boolean logic circuits.

Clarification

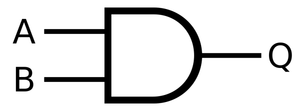
Students should be able to interpret and use symbols used in Boolean logic circuits, composed of logic gates, to determine the truthfulness of statements. Students should be able to interpret and use the following symbols:



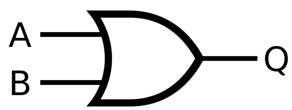
Buffer Gate
Input A
Output A



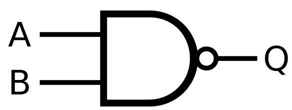
NOT Gate
Input A
Output ~A



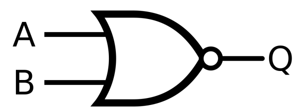
AND Gate
Inputs A,B
Output A∧B



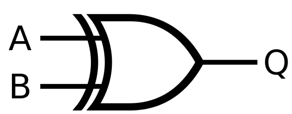
OR Gate
Inputs A,B
Output A∨B



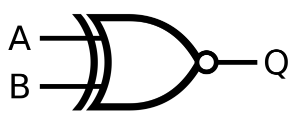
NAND Gate
Inputs A,B
Output ~(A∧B)



NOR Gate
Inputs A,B
Output ~(A∨B)



XOR Gate
Inputs A,B
Output A⊕B

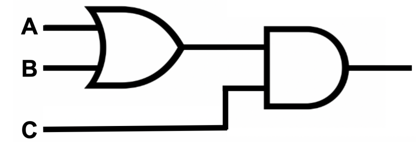


XNOR Gate
Inputs A,B
Output ~(A⊕B)

Students should interpret a “1” as being true and a “0” as being false. Students should be able to check the truthfulness of statements that include multiple gates.

Checking for Understanding

Indicator: If A = 0 and B = 1 and C=0, find whether the following statement is true (1) or false (0) and explain:



Answer: The first gate is A OR B. With A being off (false) and B being on (true), A OR B is on (true).
The second gate is (A OR B) AND C. With A OR B being on (true) and C being off (false), the second gate is off (false).

Return to: Standards

DCS.L.1 Evaluate mathematical logic to model and solve problems. DCS.L.1.4 Judge whether two statements are logically equivalent using truth tables.	
Clarification	**Checking for Understanding**
Two logic statements are equivalent if the inputs and resulting outputs are the same. For this objective, students will use truth tables to demonstrate the claim of equivalency.	**Indicator**: Use a truth table to prove the following statement. $p \rightarrow q = (q \vee \sim p)$ *Answer:*

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$\sim p$	$q \vee \sim p$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

Since the all sets of inputs return the same outputs, respectively, these two statements are logically equivalent

Return to: Standards