North Carolina Department of Public Instruction
Mathematics

Math Standards Review & Revision

NC High School Math I, II & III Courses

Dr. Maria Pitre-Martin
Chief Academic and Digital Learning Officer
Dr. Jennifer L. Curtis
Section Chief K-12 Mathematics

Policy

- **Policy ID Number:** GCS-F-012
- **Policy Title:** Policy delineating the Standard Course of Study Curriculum Development Process
  - Review data and research, surveys and other feedback;
  - Establish writing teams and develop drafts of proposed changes;
  - Submit draft for public review and input;
  - Revise draft as necessary;
  - Submit to State Board of Education for discussion and approval; and
  - Conduct professional development for teachers and administrators.

Comparing the Standards

Original Math I Standard:

A-REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Note: *At this level, focus on linear and exponential equations.*

Rewritten NC Math 1 Standard:

NC.M1.A-REI.10 Understand that the graph of a two variable equation represents the set of all solutions to the equation.

Stakeholder Feedback:

- Suggested Standard Rewording: Understand that different representations result in different graph forms each unique to its equation. The graph of each equation in 2 variables is the set of all its solutions plotted in the coordinate plane.

Teacher Survey, Math I, December 2014

Comparing the Standards

Original Math II Standard:

N-CN.9 Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials

Rewritten NC Math 3 Standard:

NC.M3.N-CN.9 Use the Fundamental Theorem of Algebra to determine the number and potential types of solutions for polynomial functions.

Stakeholder Feedback:

- "….the Fundamental Theorem of Algebra are very advanced for a second level math class."

LEA Feedback on 1st Draft Standards, Math II, April 2016

Review Policy & Procedures

February - March 2016 In accordance with SBE Policy GCS-F-012, a Data Review Committee was convened to complete data analysis and guidance for Writing Teams.

March - April 2016 Writing teams met to produce first draft

April 18 - 26 All LEAs and Charters were given opportunity to provide feedback as an extension of the writing teams

April 28, 2016 Writing teams met in Greensboro to analyze and apply feedback to create second draft

Data Used in the Process

- Development of Data Review Committee Recommendations for Revisions
  - ASRC Final Report December 2015
  - Teacher regional Focus Groups, Teacher, parent & community Surveys
- Writing Teams
  - Data Review Committee Recommendations for Revisions
  - LEA Feedback from First Draft NC Math 1, NC Math 2 & NC Math 3

Specifics of Data Used by Data Review Group

Teacher Survey	3314 Teacher Responses K-Math III 591 Teacher Responses Math I, II & III 2014
Community Survey	January - April 2015
Teacher Focus Groups	Held in 8 Regions, every LEA Represented Over 200 teachers by grade band 2014
Leader Focus Group	NCCTM (NC Council Teachers of Mathematics) Leader Session Focus Group - over 150, 2015
Academic Standards Review Commission	Final Report Submitted December 31, 2015

Overall Process Continued

- April 18 - April 26, 2016 LEA received first draft
- April 26, 2016 Feedback due (qualitative attached)
- April 28 - May 2, 2016 Feedback analyzed and applied to inform second draft

<u>Today & Moving Forward</u>

- May 4, 2016 SBE receives draft & feedback
- May 4 – 20, 2016 Public Survey window
- May 20 – May 31 Feedback cycle repeated
- June 1-2, 2016 SBE Action Item
- *June 6 - July 30, 2016 Regional meetings to communicate revisions

* Pending SBE approval

Brief Overview of Results

Quantitative data from LEA Surveys

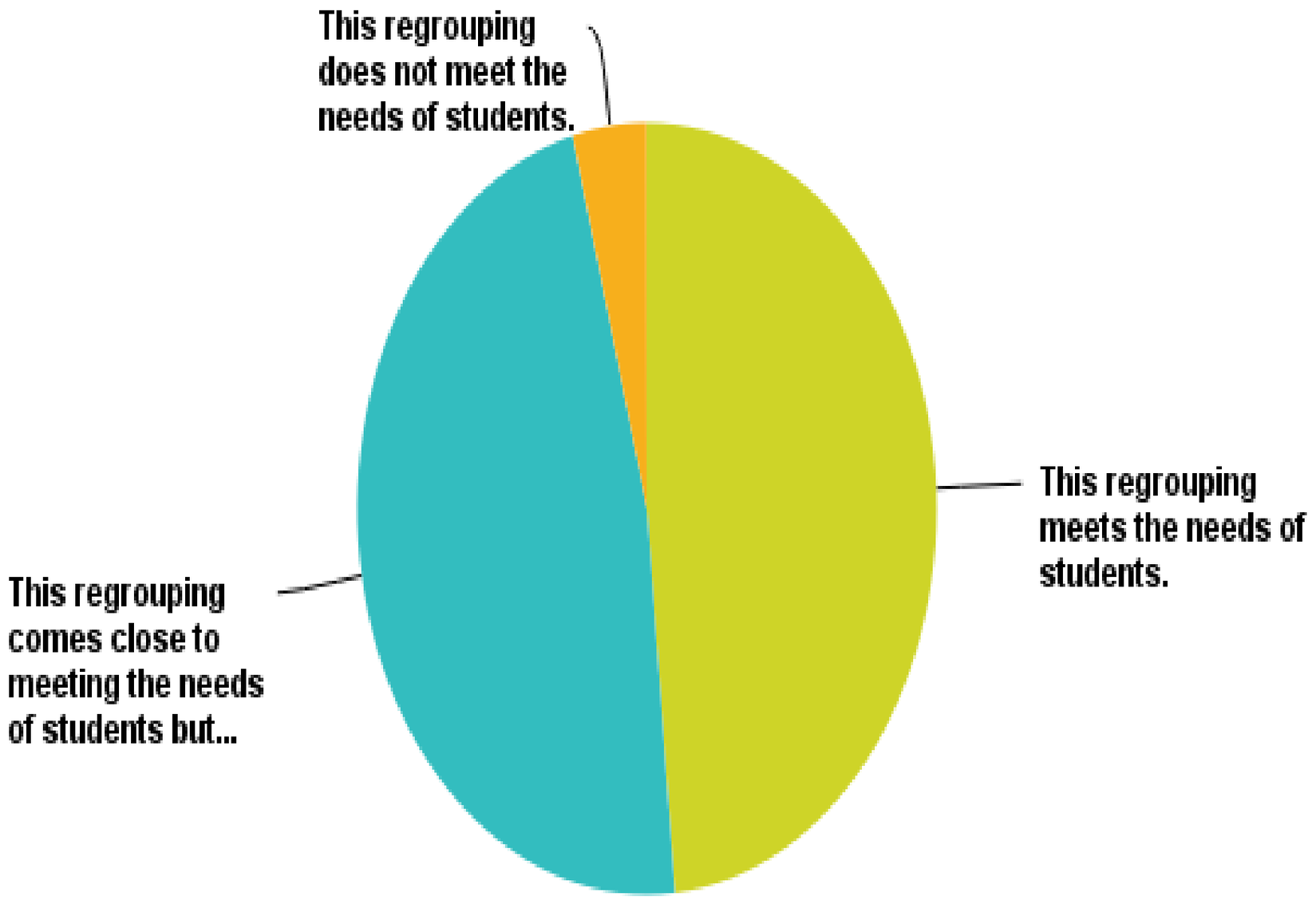
86 of 115 LEAs completed survey
for a response rate of 75%

LEA Feedback Overview

** 93 includes duplicate surveys

Answer Choices	Responses	
LEA	95.88%	93
Charter	4.12%	4
Total		**97**

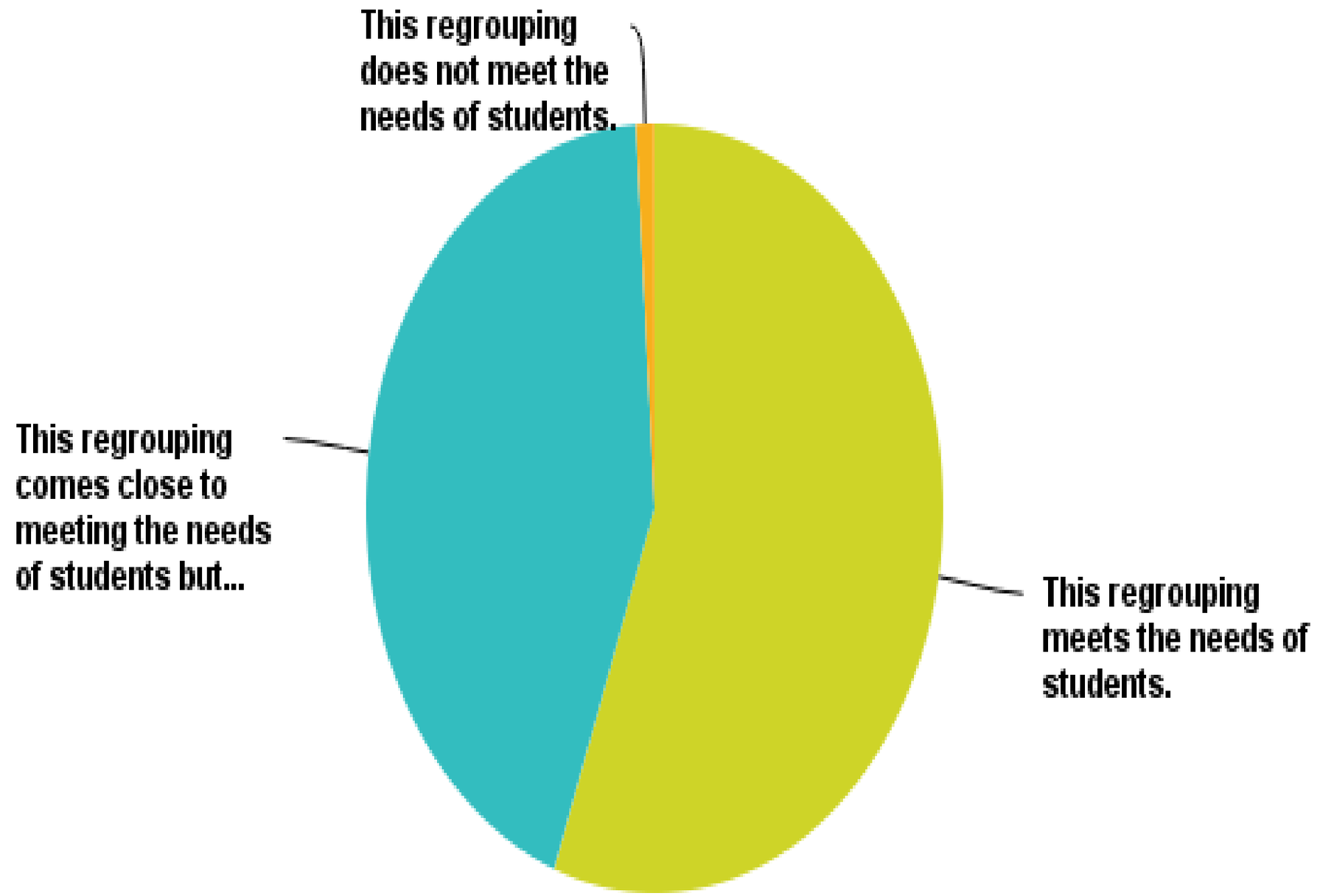
The Geometry standards were regrouped by course according to the following advice that came from teachers across the state and the Academic Standards Review Commission: Which statement best represents your team's opinion of this regrouping?



The Geometry standards were regrouped by course according to the following advice that came from teachers across the state and the Academic Standards Review Commission: Which statement best represents your team's opinion of this regrouping?

Answer Choices	Responses	
This regrouping meets the needs of students.	**48.45%**	47
This regrouping comes close to meeting the needs of students but still needs revisions.	**47.42%**	46
This regrouping does not meet the needs of students.	**4.12%**	4
Total		**97**

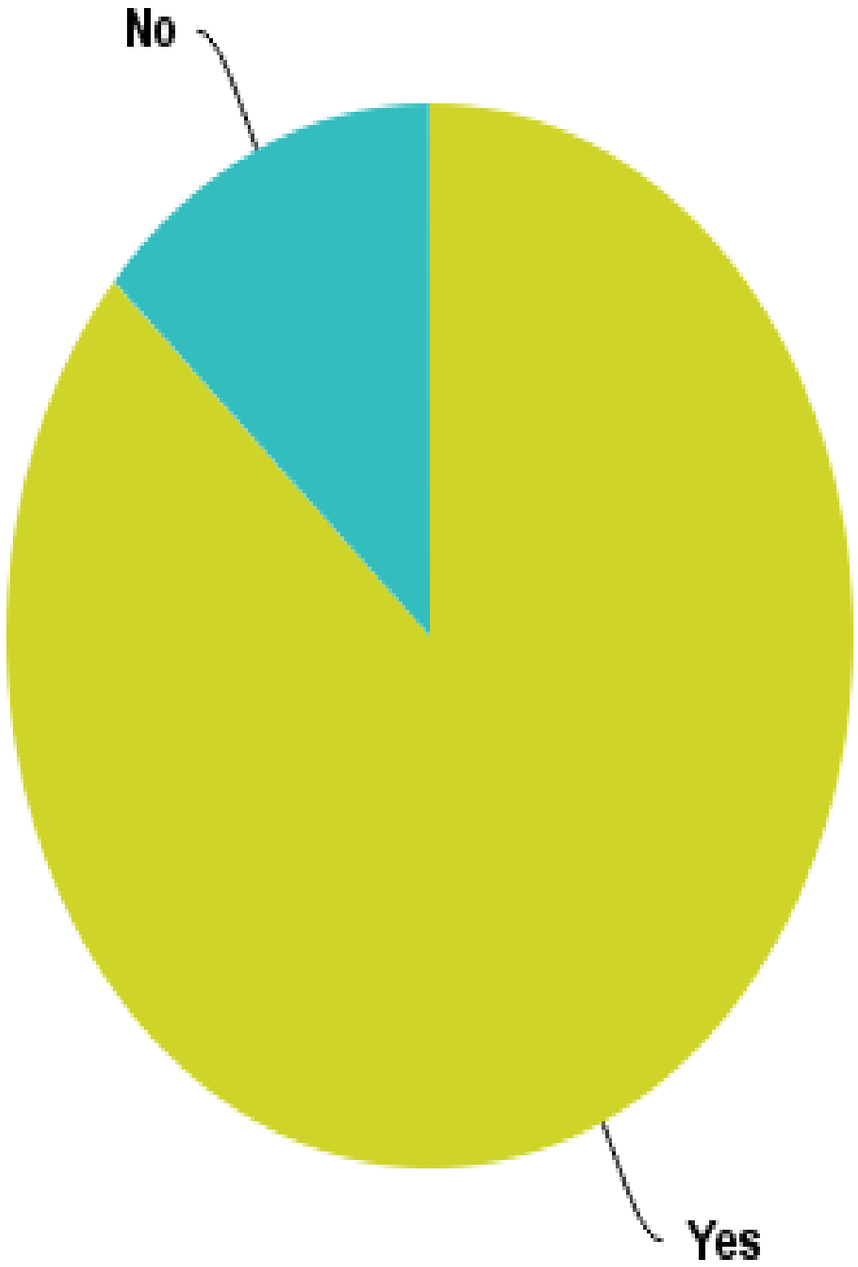
The Algebra standards were regrouped by course according to the following advice that came from teachers across the state and the Academic Standards Review Commission: Which statement best represents your team's opinion of this regrouping



The Algebra standards were regrouped by course according to the following advice that came from teachers across the state and the Academic Standards Review Commission: Which statement best represents your team's opinion of this regrouping?

Answer Choices	Responses	
This regrouping meets the needs of students.	**55.67%**	54
This regrouping comes close to meeting the needs of students but still needs revisions.	**43.30%**	42
This regrouping does not meet the needs of students.	**1.03%**	1
Total		**97**

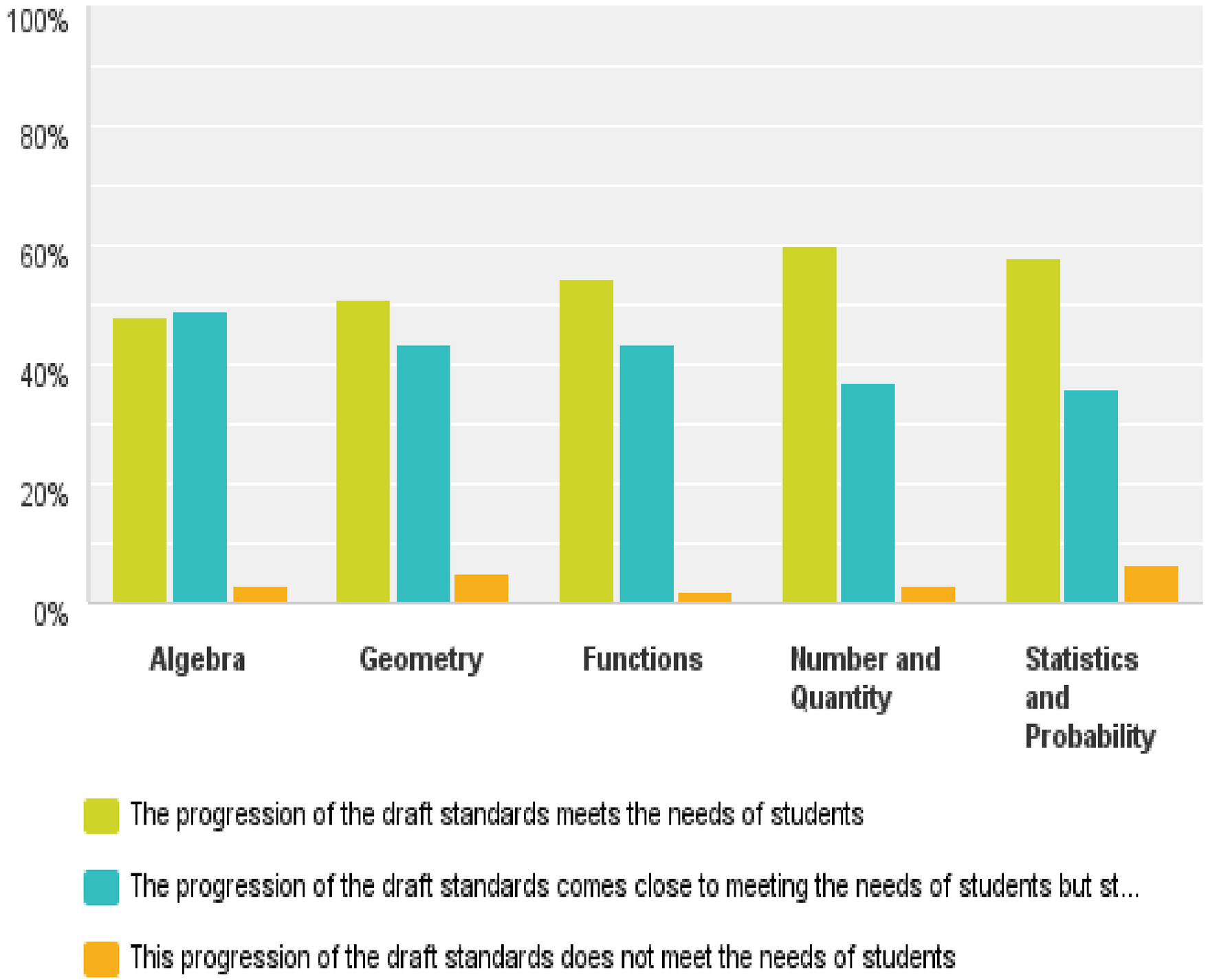
In looking at the 2016 draft standards for NC Math 1, NC Math 2 and NC Math 3 high school, did the standards provide greater clarity than the previous standards known as Math I, Math II and Math III?



In looking at the 2016 draft standards for NC Math 1, NC Math 2 and NC Math 3 high school, did the standards provide greater clarity than the previous standards known as Math I, Math II and Math III?

Answer Choices	Responses	
Yes	**86.60%**	84
No	**13.40%**	13
Total		**97**

Which statement best represents your team's opinion of the progression of the draft standards in Math I, II and III for each conceptual category



Which statement best represents your team's opinion of the progression of the draft standards in Math I, II and III for each conceptual category

	The progression of the draft standards meets the needs of students	The progression of the draft standards comes close to meeting the needs of students but still needs revisions	This progression of the draft standards does not meet the needs of students	Total
Algebra	**47.92%** 46	**48.96%** 47	**3.13%** 3	96
Geometry	**51.04%** 49	**43.75%** 42	**5.21%** 5	96
Functions	**54.26%** 51	**43.62%** 41	**2.13%** 2	94
Number and Quantity	**60.00%** 57	**36.84%** 35	**3.16%** 3	95
Statistics and Probability	**57.89%** 55	**35.79%** 34	**6.32%** 6	95

LEA Feedback – Extension of Writing Teams

Qualitative Data Solicited via three open-ended Questions *

1) Suggestions for revision

2) What is your professional opinion on the draft standards providing focused direction for teachers in each course by limiting the standards that are repeated in multiple courses?

3) Additional comments

* Attachments 5, 6 and 7

Treatment of Data

- **Analyze** … consider and categorize feedback
- **Interpret** … make recommendations for revision by course and category
- **Report** out whole group from committees and reach consensus on recommendations for revision of draft 1 to produce draft 2
- **Apply** working groups applied consensus to produce draft 2 in writing teams for FAQ, Rationale and standards.

April 28 Agenda to Apply Feedback

TIme	Activity	Goal	Notes
8	Introductions		
8:15-8:45	Individual Read	Review feedback	
8:45 - 9:00	Break	Get it out so we can work	
9:00-10:00	**Analysis Groups** 1. Big Ideas (No Judgment) 2. Identifying Priorities (Applying judgment) 3. Companion Documents 4. Coherence Concerns	Consider, categorize, and interpret feedback	
10:00-11:30	**Interpretation Groups** 1. NC Math I 2. NC Math 2 & 3 3. Number/Algebra/Functions 4. Geometry	Make recommendations for revision	A copy of the Current Standards and the 1st Draft can be found here. (These are view and copy only documents.)
11:30 - 12:00	**Reporting Recommendations**	Agree on recommendations for revisions	No verbal comments -- all in parking lot Available for afternoon working groups
	Lunch		
12:00-3:00	**Working Groups** 1. Standards Groups a. Number/Algebra/Functions	Applying recommendations	Geo - Tim, Sonia, Lisa AFN - Stef, Holt, Joe

Collaboration for the Second Draft

This team was working on editing the summary and overview of the new courses and tracking moved and integrated standards

Mike Jones (Nash-Rocky Mount), Katie Mawhinney (Appalachian State), Velvet Simington (Winstom-Salem Forsyth), Michelle Stephan (UNC-Charlotte).

This team is incorporating feedback from the LEAs to revise the 1st draft of the standards for Number & Quantity, Algebra and Functions.

Joe Reaper (DPI Consultant), Stefanie Buckner (Buncombe County), Holt Wilson(UNC – Greensboro)

.

Big Ideas and Themes (with no judgment)

Math II has grown bigger.
Questions regarding matrices introduced in Math I and not applied or used anywhere else.
Concerned with the statements that this has been moved to another course--where? In a fourth course? What is the fourth course?
Concern about normal distribution and standard deviation removed from Math III.
More clearly defined expectation for depth in courses with particular function families (particularly when a function family is distributed across multiple courses).
Unclear whether the same function families are included in the algebra and function conceptual categories (at all course levels).

Big Ideas and Themes (with no judgment)

Some concern about complex numbers in Math II.
Question including Law of Sines and Law of Cosines in Math II.
Concern about how much geometry is included.
Question whether enough geometry is included prior to taking the ACT/SAT
More clarity on direct, inverse, and compound variation across all three courses
Concern about limitations on the denominator of rational expressions (linear)
Clarify that all functions prior to Math III are included in Math III
Concern about limiting logarithmic bases to 10 and e

Draft Standards

Current Standard Abbreviation	Current Standard	Proposed Standard Abbreviation	First Draft Proposed Standard	Second Draft Proposed Standard
A-REI.4 A-REI.4a A-REI.4b	Solve equations and inequalities in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. ***Note:*** *Solve quadratic equations that have real solutions and recognize quadratic equations that do not have a real solution. (Writing complex solutions is not expected in Math II.)*	NC.M2.A-REI.4 NC.M2.A-REI.4a NC.M2.A-REI.4b	Solve equations and inequalities in one variable. a. Rewrite a quadratic equation in vertex form, using the method of completing the square. Show that the algebraic methods for solving quadratic equations, factoring, completing the square and the quadratic formula produce the same solutions. Explain why this is true using mathematical reasoning. b. Solve quadratic equations in one variable by factoring, completing the square, and the quadratic formula. Explain when quadratic equations will have non-real solutions, through graphs and the discriminant. Express non-real solutions as complex numbers.	Solve for all solutions of quadratic equations in one variable. a. Understand that the quadratic formula is the generalization of solving $ax^2 + bx + c$ by using the process of completing the square. b. Explain when quadratic equations will have non-real solutions and express complex solutions as $a \pm bi$ for real numbers a and b.
Reasoning with Equations and Inequalities ***Solve systems of equations.***				
A-REI.7	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.	NC.M2.A-REI.7	Find the solution to a system of two equations, graphically and algebraically, in two variables that consists of a(n): • Linear equation and a constant, linear, quadratic, or radical equation. • Quadratic equation and a constant, linear, or quadratic equation. • Radical equation and a constant or linear equation. • Inverse variation and a constant or linear equation.	Use tables, graphs, and algebraic methods to approximate or find exact solutions of systems of linear and quadratic equations, and interpret the solutions in terms of a context.
Reasoning with Equations and Inequalities ***Represent and solve equations and inequalities graphically.***				
A-REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	NC.M2.A-REI.10	Understand that for equations with two variables, x and y, all points, (x, y), on the graph of equation are solutions to that equation. At this level, extend to radical and inverse variation equations.	*After the 1st draft, this standard was removed from Math 2. Students should continue with the learning from Math 1 to apply to Math 2 standards.*

Next Steps - Revision

Public Comment immediately following meeting
May 4 – May 20, 2016

Same process… analyze feedback and revise as appropriate

Continue working on resource documents

Tentative Schedule has been developed for summer sessions during June and July in all 8 regions

Next Steps - Ongoing Support

In order to support teachers, a comprehensive Professional Development plan is underway.

Both internal and external partners will be involved.

Vision involves a fluid resource that is available online with print versions.

Will address revisions as well as FAQ and commonly asked for resources that are OER

Testing Thoughts

Testing/Accountability and the Math Curriculum & Instructional teams will review drafts against current forms of Math I EOC and Math II & III Final Exams.

If SBE adopts revised standards for 2016-17, then NC Math 2 & NC Math 3 Final Exams will be in a field test year.

NC Math 1 is TBD until final draft is produced.

K-8 Timeline 2016
Will be adding Fourth Level Courses

Date	Action	Description
	2016	
September-October	*K- 8 Math Data Review Committee Meets* *Fourth level maths: Advanced Functions and Modeling (AFM), Discrete Math, Pre-Calculus & SREB Essentials College Math Review Committees*	Group will conduct a thorough analysis of all feedback from teacher surveys, community surveys, teacher focus groups, leader focus group, and the ASRC .
December-January	*Math Data Review Committee findings compiled and shared*	Updates presented at December 2016 and January 2017 SBE meeting. Results from September-October Data Review Committee meetings shared including how the results and data from ASRC, parent and community survey, teacher survey, math leader focus group and teacher focus groups were organized and prioritized for K-8 to identify priorities, concerns and changes.

K-8 and Fourth Level Math Timeline 2017

2017		
January-February	*Writing Teams Convene*	Writing teams convene to create drafts of K-8 standards.
March	*Drafts K – 8 Released* *Fourth Maths*	K-8 Math drafts released to public for comments and shared with SBE.
April	*Update on Drafts* *Fourth Maths for discussion*	Update provided to SBE on data collection from public comment.
May	*K-8 Math Drafts Presented for discussion* *Fourth Maths for Action*	Final drafts with comments shared with SBE.
June	*K-8 Math Drafts Action*	Present K-8 Math standards draft for action.
June - Aug	*Regional PD sessions*	Host regional professional development and information sessions on revisions to K-8 Math standards by grade bands of K-2, 3-5, and 6-8 for teachers and district leadership.
Aug	*Implement Standards*	Districts implement new standards. Continue support.

May I answer questions that you have?

Follow Us!

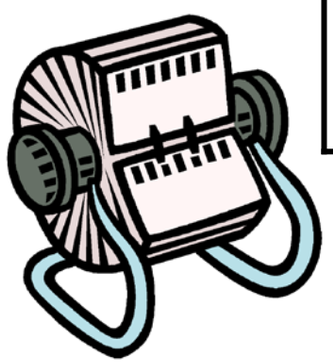
NC Mathematics
www.facebook.com/NorthCarolinaMathematics

@ncmathematics

http://maccss.ncdpi.wikispaces.net

DPI Mathematics Section

Kitty Rutherford Elementary Mathematics Consultant 919-807-3841 kitty.rutherford@dpi.nc.gov	**Denise Schulz** Elementary Mathematics Consultant 919-807-3839 denise.schulz@dpi.nc.gov
Lisa Ashe Secondary Mathematics Consultant 919-807-3909 lisa.ashe@dpi.nc.gov	**Joseph Reaper** Secondary Mathematics Consultant 919-807-3691 joseph.reaper@dpi.nc.gov
Dr. Jennifer Curtis K – 12 Mathematics Section Chief 919-807-3838 jennifer.curtis@dpi.nc.gov	**Susan Hart** Mathematics Program Assistant 919-807-3846 susan.hart@dpi.nc.gov



Summary, Overview, and Rationale for 2016 Revisions of NC Math I, II and III to NC Math 1, 2, and 3

Draft: May 3, 2016

The High School Standards Revisions Writing Groups considered the work and recommendations of the *Academic Standards Review Commission*, teachers' comments from surveys and focus groups and the recommendations of the High School Data Review Committee in reviewing the standards related to number and quantity, algebra, functions, geometry and statistics & probability in the North Carolina Standard Course of Study. Overwhelmingly, all stakeholder groups who contributed comments on the standards requested clarification and a more coherent placement of standards; this to include course level clarification on

- the specific families of functions for focused study in the Algebra and Function strands;
- standards repeated across the three high school courses; and
- the wording and articulation of standards, to include the removal of non-applicable examples and notes.

The decision to move from a traditional sequence of Algebra 1, Geometry and Algebra 2 to the integrated approach allowed North Carolina to create opportunities for teachers and students to make mathematical connections throughout high school. There is now an opportunity for algebra skills to be used every year by continuing the integrated courses from K-8 through the first 3 maths in high school. This integrated organization of standards provides the same opportunities for advanced work in mathematics during high school, as does the Algebra 1, Geometry, Algebra 2 model.

Standards for Mathematical Practice

It should be noted that the Standards for Mathematical Practice (SMP) continue to be included as the foundation for reasoning mathematically in high school. Their inclusion in each course emphasizes the importance of providing opportunities throughout ALL content standards for students to analyze, argue, model, and problem solve in meaningful ways. Mathematics is a human activity of analyzing and solving problems and finding solutions; therefore, the SMP are critical for enabling students to learn mathematics in genuine ways. Important to the reading of the revision, is the understanding that the foundation of the Standards for Mathematical Practice are still intact. Therefore, the SMPs are listed in each course.

Modeling with mathematics also remains an integral part of all of the high school courses, in all of the content domains. While modeling with mathematics is the fourth SMP, we use the definition from The Consortium for Mathematics and its Applications (COMAP) and the Society for Industrial and Applied Mathematics (SIAM) to detail more specifically the process of mathematical modeling that the SMP bring to the content standards:

Mathematical modeling is a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena.

(Guidelines for Assessment and Instruction in Mathematical Modeling Education (GAIMME), 2015)

Mathematical modeling is the way students connect the mathematical content they are learning to the real world in which they live. The vision for mathematics education in North Carolina is to ensure North Carolina students have mathematical understanding at or above the level of their national and international peers, ensuring that they are life, college and career ready.

Number and Quantity Conceptual Category

The number and quantity domain has been an area that has produced many questions and concerns. While these standards are essential to modeling and mathematics in general, the majority of the content in these standards is located throughout the Standards for Mathematical Practice. For this reason, N-Q.1, N-Q.2, and N-Q.3 will be completely integrated into the Standards for Mathematical Practice.

Focus and Rationale:

NC Math 1

In NC Math 1, students apply and extend their understanding of the number system from middle school to high school number and quantity. In 7th grade, students were formally introduced to the rational number system. In 8th grade, students were introduced to the real number system through the study of irrational numbers, approximating square and cube roots, and applying the properties of integer exponents to numerical expressions. This is significantly important to the systematic study of functions, which is a significant part of the HS standards. NC Math 1 students will apply their understanding and use of irrational numbers and square roots when solving simple quadratic equations by taking square roots. They will also apply their knowledge of the properties of integer exponents to algebraic expressions.

NC Math 2

In NC Math 2, students continue to build upon their knowledge of the properties of exponents by interpreting and using rational exponents. Students will use this knowledge to rewrite expressions with rational exponents or radicals into equivalent forms. Students will continue to investigate and formalize their understanding of the rational number system and irrational numbers. As students become fluent with quadratic equations in Math 2, they will become aware of the existence of complex numbers and use them as solutions to quadratic equations.

NC Math 3

In NC Math 3, students work with complex numbers and the Fundamental Theorem of Algebra as they study polynomial functions (quadratic and higher degree).

Algebra and Functions Conceptual Categories

The conceptual categories of Algebra and Functions are inter-related. Functions describe situations in which one quantity determines the other. The difference between the Function standards and the Algebra standards is that the Function standards focus more on the characteristics of functions, e.g., domain/range, function definition, etc. whereas the Algebra standards provide the computational tools and understandings that students need to explore specific instances of functions. As students progress through high school, the coursework with specific families of functions and algebraic manipulation evolve. For example the focus in NC Math 1 is on linear, quadratic, and exponential function families along with the algebraic concepts of solving equations, factoring quadratics, etc.

Algebra Conceptual Category

The Algebra Conceptual Category serves the purpose of connecting the generalizations of arithmetic operations with the function and geometric concepts across the standards. The learning in each course will align to the study of function types in each HS math course.

Focus and Rationale:

NC Math 1

The focus of algebra in NC Math 1 is on linear, exponential, and quadratic expressions/equations. NC Math 1 students will recognize the difference between the distinct expression types and make connections between expressions, equations, and functions. They build on the understanding of arithmetic operations. Solving equations and the basic introduction of linear equations was established in the 8th grade where students extensively studied linear and nonlinear functions. The goal of NC Math 1 is for students to develop a thorough understanding of linear functions as they begin studying exponential and quadratic equations/functions; viewing exponential and quadratic equations as extensions of nonlinear functions learned in the previous course. The focus in NC Math 1 is on these three equation types and the operations involved with them. Solving quadratic equations by factoring is now included in NC Math 1 and the appropriate related standards have been moved into the course. The quadratic formula is not an acceptable tool for factoring at the Math 1 level. This is to address a more coherent learning of quadratics in HS mathematics. It is expected that these equations come from a modeling context, when appropriate.

NC Math 2

The focus of algebra in NC Math 2 is on extending work with quadratic equations that were introduced in the previous HS course. The body of work for quadratics was previously split between the three HS math courses. The major work of quadratics has been moved to NC Math 2, including solving quadratics with complex solutions and completing the square to rewriting a quadratic in vertex form. Students will use rational exponents to rewrite radical expressions in an equivalent form. This concept has been moved from NC Math 1 to align with algebraic methods of solving radical equations. Inverse variation remains a focus of study in NC Math 2.

NC Math 3

The focus of algebra in NC Math 3 is to allow students to recognize the connection between arithmetic operations and equations. When entering Math 3, students should have mastery of linear, exponential, quadratic, and radical relationships, as well as inverse variation. Students will apply their knowledge of

these equations and functions to solving polynomials with the highest degree of 3 and solving absolute value equations and inequalities algebraically and graphically; to the relationship between exponential and logarithmic equations; and to rational equations limited to denominators with linear expressions. It is expected that these equations come from modeling contexts, when appropriate.

Functions Conceptual Category

Since elementary school, students have been working with patterns and focusing on the change from one term to the next in a sequence. In the middle grades, students learn to relate change across two patterns through ratio tables and to quantify this relationship with a rate. In grade 8, students are formally introduced to the concept of a function. They learn that a function describes a relationship between an input and output value, identify examples of function and non-functions, and develop an understanding of the family of linear functions. Students' work with function in high school should be seen as a next step in the development and refinement of their study of relationships between two varying quantities. The function families that are the focus of each course are listed in the diagram below:

NC Math 1	NC Math 2	NC Math 3
Linear	Quadratic	Exponential
Exponential	Square Root	Logarithm
Quadratic	Inverse Variation	Rationals w/ linear denominator
Functions	Functions	Polynomial w/ degree $\leq$ three
		Absolute Value and Piecewise

Note that the table is meant to show a progression so that students' understanding of each function family evolves over the three courses. In 8th grade, students are introduced to the notion of linear functions as they contrast with other functions in general. In NC Math 1, families of exponential and quadratic functions are added to the function types that students now explore. Students should become fluent in quadratic functions by Math 2, along with square root and inverse functions. Finally, in Math 3, exponential functions are revisited with logarithmic, rational, absolute value, piecewise, and polynomial functions added.

Focus and Rationale:

NC Math 1

In NC Math 1, students apply and extend their understandings of functions from grade 8 to the formal definition of a function and the use of function notation when expressing functions symbolically. They recognize that, in general, sequences defined recursively or explicitly are functions, and that arithmetic sequences are linear functions and that geometric sequences are exponential functions. They interpret and analyze different representations of a broad array of functions with a concentration on linear and exponential functions. Additionally, NC Math 1 students use recursive processes to build a variety of functions, construct and use explicit linear and exponential functions, and compare linear and exponential function models by interpreting the parameters of each model in terms of the context they represent.

NC Math 2

In NC Math 2, students apply and extend their understandings of functions from NC Math 1 by fluently operating with quadratic functions, building their repertoire of function families to include square root, and inverse functions.

NC Math 3

In NC Math 3, students use their learning about functions from previous courses to engage with a wider range of function families, including polynomial, rational, trigonometric, absolute value and piecewise-defined functions. Students find inverse functions and begin to explore the cosine and sine functions graphically. In various 4^{th} level math course options, students will have opportunities to reach fluency with the function families introduced in the course.

Geometry Conceptual Category

Included in the Geometry Conceptual Category is a 21st Century approach to the concepts of congruence and similarity. Learning geometry from a transformational point of view provides students with the 21st century tools and understanding to use their geometric knowledge in STEM-related fields from engineering to computer science to data transfer. This current positioning of geometry across the standards of NC Math 1, 2, and 3 allows us to help students build connections between their geometric understanding and mathematics as a whole.

Concepts of congruence and similarity are developed through the use of transformations (translations, rotations, reflections, and dilations) which gives students a visual and kinesthetic means of discussing and understanding geometric ideas. Given the importance of geometric ideas across the standards, The High School Data Review Committee & Writing Team recognized the need to articulate clearly where geometry topics are situated in the standards revision, why they are positioned in each course, and how they connect to the overall themes in each of the courses NC Math 1, NC Math 2, and NC Math 3.

NC Math 1	NC Math 2	NC Math 3
Focus on coordinate geometry • Distance on the coordinate plane • Midpoint of line segments • Slopes of parallel and perpendicular lines • Prove geometric theorems algebraically • Rational: Students make connections between algebra & geometry concepts via the coordinate plane	Focus on triangles • Congruence • Similarity moved from NC Math 3 to provide coherence • Right triangle trigonometry • Add special right triangles (assessed on the ACT)	Focus on circles • Move all content related to circles from NC Math 1 and NC Math 2 to NC Math 3 to provide coherence • Introduce the concept of radian • Introduce periodic functions but leave main work of trigonometric functions in 4th level math courses.
Additional Considerations		
Move the development of area and volume formulas to MS	Introduce proof • Students should use both informal and formal methods of reasoning to prove theorems related to lines, angles, and triangles	Formalize Proof • Focus on both paragraph and flow proofs
	Continue development of concepts around parallel lines and angles introduced in MS; important for proving theorems about triangles	Capstone learning with the connection of algebraic and geometric concepts through solving modeling and design problems

Constructions were removed as standards. They will be addressed in curricular resources as an instructional method since they are used to support student learning of concepts.

Developmentally, this is also appropriate—the work of Dina van Hiele-Geldof and her husband, Pierre van Hiele led to the development of *van Hiele Levels of Geometric Understanding.* These levels help us understand how students come to learn geometry. Students need incremental introduction to the topics of geometry with increasing sophistication that allows them to build on their concrete experiences with shapes and space and to identify relationships between properties of shapes. Students' ability to reason both informally and formally about the logical structure of geometry develops as they are more able to handle abstractions.

Focus and Rationale:

NC Math 1

The focus of geometry in NC Math 1 is ***coordinate geometry***. Students establish relationships and verify/prove them using coordinates and algebraic reasoning. The connections to the coordinate plane, distance, midpoint, and slope fit nicely with the emphasis on linear functions and algebraic reasoning in NC Math 1. The conceptual development of area and volume formulas will be moved to middle school when they are introduced and first used. Likewise, we will move the use of area and volume formulas to solve problems to Math 3 to support the goal of geometric modeling and design.

NC Math 2

The focus of geometry in NC Math 2 is on ***geometric relationships and properties of shape, focusing on lines, angles, and triangles***. Students continue to build on the middle school development of relationships about lines and angles to support proof of theorems about triangles. ***Triangles*** are the most fundamental two-dimensional polygon; an in-depth understanding of triangles will support the study of other polygons and circles in NC Math 3. ***Congruence*** and ***similarity*** are developed through a ***transformational approach***. Students are introduced to the ideas of ***geometric proof***, using both informal and formal methods of proof to develop and organize logical arguments. Similarity is utilized to develop ***right triangle trigonometry***, and special right triangles are introduced.

NC Math 3

The focus of geometry in NC Math 3 is on ***circles***. Students continue their study of relationships in polygons by ***conjecturing and proving relationships about quadrilaterals***. The goal in proof is to further develop the ability to construct logical arguments and to develop both ***flow*** and ***paragraph proofs***. Students develop ideas and properties about circles, including the idea of ***radians***. Periodic functions are introduced but the main work with trigonometric functions is housed in the fourth math courses. There is opportunity for ***capstone learning***— the connection of students' ideas of algebra, geometry, and functions—through solving ***modeling and design problems***.

General notes:

The writing team developed a ***list of the theorems*** that are to be proven in each course. This list is not exhaustive, but the standard. Proving theorems should include the relationships on the list, but are not limited to those alone. The expectation is that proofs will emphasize paragraph and flow proofs. The traditional 2-column proof is not the expectation. The construction of logical arguments and the ability to explain their reasoning is what will be expected from students.

Statistics and Probability Conceptual Category

In an increasingly data-driven world, statistical literacy is becoming an essential competency, not only for researchers conducting formal statistical analyses, but for informed citizens making everyday decisions based on data. Whether following media coverage of current events, making financial decisions, or assessing health risks, the ability to process statistical information is critical for navigating modern society.

(Franklin et al, *Statistical Education of Teachers*, 2014)

As informed citizens of an "increasingly data-driven world" students will need to be able to calculate useful statistics and probabilities, as well as understand what those calculations mean about the data within the given context. They will be asked to create and utilize representations and models of data. Further, students will need to be critical of the process of statistical investigation which influences the validity of sample statistics as representative of a population.

Focus and Rationale:

NC Math 1

The focus of *Statistics and Probability* in NC Math 1 includes analysis of univariate and bivariate data. With univariate data students will calculate and use statistics measuring center and spread that describe characteristics of a data set and that distinguish one data set from another. With bivariate data students will use scatterplots to study associations between variables. Regression is limited to linear and exponential functions.

Students will have created graphical representations of univariate data in 8th grade so that in NC Math 1 they will be ready to utilize technology to create such representations efficiently. Instructional focus should be placed on interpreting the characteristics of the data seen within the representation or using the representations to compare and contrast multiple data sets.

The course treatment of bivariate data includes the creation of scatterplots and the creation of functions to fit the plots. Students should be asked to consider only linear and exponential functions as potential tools for modeling the data.

Before fitting a linear function to a data set, students will describe the possible association of the data values (strongly or weakly associated, positively or negatively associated) that is seen from the scatterplot. Then when determining a linear function that fits a scatterplot, students will consider residual values as an indication of the goodness of fit of the linear function.

The American Statistical Association's *GAISE Report* (2007) places the understanding of Pearson's correlation coefficient as a tool for students who are working within the most sophisticated level of school statistics (there are 3 levels in the framework provided in the report). Analyzing residuals is a first step toward understanding Pearson's correlation coefficient as a tool for determining goodness of fit, thus supporting students' development toward that highest level within the framework.

An important part of the analysis of bivariate data and the potential association of the variables is to distinguish any observed association from causation. This requires students to consider data in its context to determine if causation is reasonable or not.

A final note about calculation and technology-
Students will benefit from calculating some number of standard deviations and residuals by hand, but technology should also be used for more efficient calculations in order to move the instructional focus to an analysis of the results of those calculations. Students should use technology to find equations of least squares regression lines or least squares exponential curves.

NC Math 2
The focus of *Statistics and Probability* in NC Math 2 is on probability. Students will compare and contrast experimental and theoretical probabilities, and within and without contexts investigate the concepts of independent events and conditional probabilities. Students will develop their understanding of the concepts of independent events and conditional probabilities, as well as apply rules of calculating probabilities that include conditional probabilities and independent events. Students will also use two-way tables of categorical data to calculate probabilities.

The *Statistics and Probability* standards for NC Math 2 include the majority of the content of probability for secondary mathematics. It is important for teachers to note that students experienced probability content in Grade 7 math, where the experience included work in both theoretical and experimental probabilities. It is in NC Math 2 that students should use the language of probability.

Several rules of probabilities are to be utilized by students and can be verified with examples. These calculations should be connected back to the context of the problem.

NC Math 3
The focus of *Statistics and Probability* in NC Math 3 is on the use of sample data to represent a population. Important understandings include the necessity of randomization in sampling to ensure an unbiased representation of a population. Students will engage in simulation to create a random sample of a population, calculating sample statistics representative of population statistics. Students will examine and evaluate the design and results of population studies that utilize samples.

Students need to be informed about the appropriate design and implementation of studies that utilize sample statistics to represent a population. This course will introduce students to the process of inference: that is, drawing conclusions about a population based on information from a sample. Students will use simulation to see that the variability in samples can be used to predict a margin of error. This work will be based on simulation only, not on any computational formulas. By the end of the course students should understand how statistics may be used to analyze data and make decisions.

References

Franklin, C., Kader, G., Mewborn, D., Moreno, J., Peck, R., Perry, M., & Scheaffer, R. (2005). Guidelines for Assessment and Instruction in Statistics Education (GAISE): A Pre-K-12 Curriculum Framework. Endorsed by the American Statistical Association (ASA).

Garfunkel, S., & Montgomery, M. (Eds). (2015). Guidelines for Assessment & Instruction in Mathematical Modeling Education (GAIMME). Funded and published jointly by Consortium for Mathematics and Its Applications (COMAP) and Society for Industrial and Applied Mathematics (SIAM).

Tarr, J., Grouws, D. Chávez, O. & Soria, V. (2013).The effects of content organization and curriculum implementation on students' mathematics learning in second-year high school courses**.** *Journal for Research in Mathematics Education*, 44(4), 6.